

Analyzing students' mathematical concept understanding in terms of their learning independence

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Abstract

One of the objectives of learning mathematics is to understand mathematical concepts. A good understanding of mathematical concepts can support the development of problem-solving skills, communication, representation, reasoning, connection, creative thinking, and critical mathematical thinking. The difficulty of understanding and applying the concepts obtained in the previous material affects the material at the next level. A shared understanding of mathematical concepts can be caused by learning independence. This study aims to analyze the group of students' mathematical comprehension ability in terms of learning independence. This research is research with descriptive methods and qualitative approaches. The subjects of this study were class XI MIPA 4 students of SMAN 2 Bantul, totaling 12 people, to examine the category of learning independence. Then purposive sampling techniques were used to take four samples from four types of learning independence. The instruments developed and used are the learning independence questionnaire and mathematical comprehension test. The results showed that subjects who had to learn independence with a high category could understand mathematical concepts by 88% and were in the high class. In comparison, students with a moderate level of learning independence could understand mathematical concepts by 42% and be in a low category.

Keywords: junior high school, learning independence, understanding mathematical concepts

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INTRODUCTION

The life skills of each individual are necessary in the development of today's world, one of which is the mathematical prowess that grows through the subject of mathematics. One of the objectives of learning mathematics, according to the Minister of Education and Culture Number 59 of 2014 concerning the 2013 SMA/MA Curriculum, is to understand the concept of mathematics. Mathematical concepts are competent in explaining the relationship between concepts and using concepts and algorithms in a flexible, accurate, efficient, and precise manner, solving problems and carrying out algorithms or procedures (Permendiknas, 2014). The concept of understanding is a crucial aspect of learning (Kurniawati & Nita, 2018; Santrock, 2018). The ability to understand mathematical concepts makes students understand the concepts learned, and the material taught is not just rote memorization (Maharani, Hartono, & Hiltrimarti, 2013). Mathematical concepts that are not yet understood will make students difficult in the learning process (Isnaini Handayani & Afifah Fitria Ramadhani, 2020). Mathematical understanding also supports problem-solving skills, reasoning, connection, communication, representation, critical thinking, and creative thinking (Hendriana, Rohaeti, & Sumarmo, 2017; Santrock, 2018; Yanti, Laswadi, Ningsih, Putra, & Ulandari, 2019).

Mathematical comprehension can be measured using indicators of understanding of concepts. Indicator of understanding mathematical concepts, i.e., being able to: 1) Re-state a concept; 2) Classify mathematical objects according to specific properties; 3) Give examples and

not examples of a concept; 4) Present concepts with a wide variety of mathematical representations; 5) Develop acceptable terms or necessary conditions of a concept; 6) Choose, use, and utilizing specific procedures; 7) Applying problem-solving concepts (Budiarti, Purwanto, & Hendriana, 2019; Haris & Jihad, 2012). The indicators are simplified to 1) Classify mathematical objects according to specific properties; 2) Select, use, and utilize specific procedures; 3) Apply the concept of problem-solving (Rusfiana & Roesiana, 2019; Suraji, Maimunah, & Saragih, 2017). In this study, indicators that have been simplified by Rusfiana, et al., and Suraji, et al.

Based on interview data, we know that students at SMA N 2 Bantul have difficulty learning, namely the problem of applying the concepts obtained in the previous material to the material to study at the next level, especially in trigonometry. Based on the results of research conducted in class X IPS 2 SMAN 17 Batam, it shows that the ability to understand mathematics in trigonometric material is still low because students only memorize formulas but cannot use that in solving the given problems (Derfia, Gusmania, & Hanggara, 2020). In addition, the standard mathematical understanding caused by the learning independence of students, understanding concepts, and learning independence are also interrelated (Belanisa, 2019; Marlina, Nurjahidah, Sugandi, & Setiawan, 2018; Muhamad, 2018; Yuliana, Surahmat, & Fathani, 2021). Students with good learning independence will be responsible for learning, which will impact learning outcomes (Arifin & Herman, 2018). Research conducted on students of class VII MTs Sunan Kalijogo Ngandari obtained conclusions: 1) subjects with high category learning independence can understand mathematical concepts with high categories, 2) subjects with medium learning independence can understand mathematical concepts are moderate, 3) subjects with independence to learn categories low can understand mathematical concepts with low categories (Prastiyo, Alifiani, & Hasana, 2021).

Independent learning conducted by learners gives freedom to learners to discover how academic life will suit daily life (Johnson, 2009). In line with Johnson's opinion, Hidayati & Listyani detailed the indicators of learning independence, namely: 1) Independent of others; 2) Have confidence; 3) Behave in a disciplined manner; 4) Have a sense of responsibility; 5) Behave on one's initiative; 6) Exercise self-control (Hidayati & Listyani, 2010).

Based on the description above, this research differs from the previous one, which was conducted in class XI MIPA 4 SMAN 2 Bantul. This research aims to analyze the level of students' mathematical understanding ability in terms of learning independence.

RESEARCH METHOD

This research is qualitative descriptive research. The subjects in this study were students of class XI MIPA 4 SMAN 2 Bantul. As many as 12 students examined the category of learning independence, then used purposive sampling techniques to take four samples from four categories of learning independence. It is namely based on the high category of one person, medium one person, low one person, and very low one, which will then study the ability to understand mathematical concepts.

Table 1. Learning Independence Criteria

Ability Level	Range of Values
High	$76 < x \leq 95$
Moderate	$57 < x \leq 76$
Low	$38 < x \leq 57$
Very Low	$19 \leq x \leq 38$

Modified from (Rosmawati & Sritresna, 2021)

x = each student's learning independence score

The instruments in this study are a questionnaire on learning independence and a test of understanding mathematical concepts. The learning independence questionnaire consists of 19 statements based on learning independence indicators developed by Hidayati and Listiyani. Researchers categorized the samples based on the level of learning independence divided into four classes as in Table 1.

The mathematical concept understanding test consists of 2 questions about the indicators of mathematical understanding simplified by Rusfiana and Roesdiana. Guidelines for scoring students' mathematical concept comprehension ability are carried by referring to the following scoring rubrics (See Table 2).

Table 2. Mathematical Comprehension Ability Scoring Rubric

Indicator	Description	Score
Classifying mathematical objects according to certain properties	Don't give an answer	0
	Incapable of classifying mathematical objects according to certain properties	1
	Able to classify mathematical objects according to certain properties, but there are many errors	2
	Able to classify mathematical objects according to properties, but there are few errors	3
	Able to classify mathematical objects according to certain properties correctly	4
Choose, use, and utilize certain procedures	Don't give an answer	0
	Unable to select, use, and utilize certain procedures	1
	Able to choose, use, and take advantage of certain procedures, but there are many errors	2
	Able to choose, use, and take advantage of certain procedures, but there are slight errors	3
	Able to choose, use, and utilize certain procedures correctly	4
Applying the concept of problem-solving	Don't give an answer	0
	Unable to apply the concept of problem-solving, but not appropriate	1
	Able to apply problem-solving concepts, but there are many errors	2
	Able to apply the concept of problem-solving, but there are a few errors	3
	Able to apply problem-solving concepts correctly	4

Modified from (Rusfiana & Roesiana, 2019)

The mathematical concept comprehension ability test data is processed by giving an assessment based on the scoring rubric and calculating the percentage of mathematical concept comprehension ability; and then the level of mathematical concept comprehension ability is determined based on Table 3.

Table 3. Categorization of Mathematical Concept Comprehension Ability

Category	Percentage
High	$75\% < P \leq 100\%$
Moderate	$50\% < P \leq 75\%$
Low	$25\% < P \leq 50\%$
Very low	$0\% \leq P \leq 25\%$

Modified from Rusfiana & Roesiana (2019)

RESULTS AND DISCUSSION

Learning independence

The results of the analysis of learning independence questionnaire data containing 19 question items carried out to 12 subjects obtained an overview as in Table 4.

Table 4. Results of the Recapitulation of the Learning Independence Questionnaire

Subject	Total Score	Ability Level
SS	87	High
To	77	High
MW	83	High
SD	81	High
HM	73	Moderate
DP	84	High
Of	84	High
Nor	67	Moderate
AA	73	Moderate
HF	62	Moderate
AN	62	Moderate
AR	64	Moderate

Based on table 4, we can know that SS has the highest learning independence with a score of 87. High learning independence is also possessed by DP, AV, MW, SD, and AI. Meanwhile, HM has moderate learning independence with a score of 73. Learning independence is also possessed by AA, NI, AR, AN, and HF. Based on the learning independence data, a sample was selected to analyze the ability to understand mathematical concepts with a purposive sampling technique, namely SS for high learning independence ability, while learning independence ability is chosen by HM.

Mathematical understanding

Based on the results of the learning independence questionnaire, a mathematical comprehension ability test was carried out on SS and HM. The results of the student's mathematical ability test obtained the following results (See Table 5).

Table 5. Student Mathematical Concept Understanding Test Results

Subject	SS	HM
Indicator 1: Classifying mathematical objects according to certain properties	7	4
Indicator 2: Choose, use, and utilize certain procedures	7	4
Indicator 3: Applying the concept of problem-solving	7	2
Total score	21	10
Percentage	88	42
Category	High	Low

Based on table 5, the ability to understand individual mathematical concepts for high and moderate levels of learning independence can be explained as follows.

Understanding of mathematical concepts in students with high category learning independence

Based on the results of the mathematical concept comprehension ability test, subjects who have a high level of learning independence, namely SS, also have a high understanding of mathematical

concepts. This can be seen from the percentage of understanding of mathematical concepts obtained, which is 88%. The results of the work of subjects who have high independence are presented in the following Figure 1.

1. Diberikan persamaan trigonometri $\sin 3x - \sin x = 0$. Persamaan tersebut memiliki penyelesaian $x = -k \cdot 2\pi$ atau $x = \frac{\pi}{4} (1+2k)$. Apakah penyelesaian tersebut benar? Berikan penjelasannya.

Untuk $x = -k \cdot 2\pi$, salah karena yang benar penyelesaiannya sebagai berikut:

► $\sin 3x - \sin x = 0$
 $\sin 3x = \sin x$
 $3x = x + k \cdot 2\pi \Rightarrow$ Jadi jawaban yang benar adalah $x = k\pi$
 $3x - x = k \cdot 2\pi$
 $2x = k \cdot 2\pi$
 $x = k\pi$

► Untuk $x = \frac{\pi}{4} (1+2k)$, benar. Penjelasan pembuktian sebagai berikut:

$3x = (\pi - x) + k \cdot 2\pi$
 $3x + x = \pi + k \cdot 2\pi$
 $4x = \pi + k \cdot 2\pi$
 $x = \frac{\pi + k \cdot 2\pi}{4} \Rightarrow$ Jadi $x = \frac{\pi}{4} (1+2k)$ adalah benar.
 $x = \frac{\pi}{4} (1+2k)$

(a)

2. Tentukan himpunan penyelesaian trigonometri persamaan $\sec^2 x + \frac{2}{\cot x} - 4 = 0$ untuk $0 \leq x \leq 2\pi$.

► $\sec^2 x + \frac{2}{\cot x} - 4 = 0$
 $\sec^2 x + 2 \tan x - 4 = 0$
 $\tan^2 x + 1 + 2 \tan x - 4 = 0 \rightarrow \tan^2 x + 2 \tan x - 3 = 0$
 $(\tan x - 1)(\tan x + 3) = 0$

$\rightarrow \tan x - 1 = 0$ $k=0 \rightarrow x = \frac{\pi}{4}$ $\rightarrow \tan x + 3 = 0$
 $\tan x = 1$ $k=1 \rightarrow x = \frac{\pi}{4} + \frac{4\pi}{4}$ $\tan x = -3$
 $\tan x = \tan 45^\circ$ $= \frac{3}{4} \pi$ (TM)
 $x = \frac{\pi}{4} + k \cdot \pi$

(b)

Figure 1. Results of the SS Mathematical Concept Comprehension Ability Test (a) question number 1 (b) question number 2

The following are the results of the analysis of understanding the mathematical concepts of each indicator based on the results of SS work:

The ability to classify mathematical objects according to a certain nature

Based on question number 1, for indicators of the ability to group/classify objects according to a certain nature, the SS answered correctly, but there was still a slight error in which the SS directly wrote down the conclusions and then wrote down the correct solution. To determine the correct solution, it is necessary to explain that the equation is a basic trigonometric equation that has two solutions. As for question item number 2, the ability to group/classify objects according to certain properties SS answers correctly, namely SS uses trigonometric identities to change trigonometric equations in the form of squares.

Ability to select, use, and utilize specific procedures or operations

Based on question number 1, for indicators of the ability to select, use and utilize certain procedures, the SS answered correctly. However, question number 2 for this indicator SS

answered correctly, but there was still a slight mistake, namely not doing the reasoning to change in the form of squares but directly working on it.

Ability to apply problem-solving concepts

Based on question number 1, on the indicators applying the concept of problem-solving SS answers correctly. The SS solved the problem by applying the concept of basic trigonometric equations to the angle of radian size. In addition, based on question number 2, in this indicator, the SS answered correctly, but there was a slight error, namely not writing down the set of settlements.

Understanding mathematical concepts of students with low category learning independence

Based on the results of the mathematical concept comprehension ability test, subjects who have a moderate level of learning independence, namely HM, have a low understanding of mathematical concepts, with a percentage of understanding mathematical concepts obtained, which is 42%. The results of the work of subjects who have high independence are presented in the following Figure 2.

1. $\sin 3x - \sin x = 0$
 $2 \cos \left(\frac{3x+x}{2} \right) \sin \left(\frac{3x-x}{2} \right) = 0$
 $2 \cos(2x) \sin(x) = 0 : 2$
 $\cos(2x) \cdot \sin(x) = 0$
 $\Rightarrow \cos(2x) = 0$ atau $\sin(x) = 0$ $k = 0, \pm 1, \pm 2$
 $2x = \frac{\pi + k \cdot \pi}{2}$ $x = k \cdot \pi$
 $x = \frac{\pi + k \cdot \pi}{4}$
 $x = \frac{\pi}{4} (1 + 2k)$ $x = -k \cdot \pi$
 karena kelipatan π positif/negatif memenuhi $\sin x = 0$

(a)

2. $\sec^2 x + \frac{2}{\cot x} - 4 = 0$
 $\tan^2 + 1 + 2 \tan x - 4 = 0$
 $\tan^2 + 2 \tan x - 3 = 0$
 $(\tan x + 3)(\tan x - 1) = 0$
 $\tan x = -3$ $\sqrt{\tan x = 1}$
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

(b)

Figure 2. Results of the HM Mathematical Concept Comprehension Ability Test (a) question number 1 (b) question number 2

The following is the result of the analysis of understanding the mathematical concepts of each indicator based on the results of HM work.

The ability to classify mathematical objects according to a certain nature

Based on question number 1, for the indicator of the ability to classify /group objects according to a certain nature. HM answers to classify objects not according to the concept. The ability to

classify objects that HM should do is to solve with trigonometric equations, but HM uses the trigonometric angle difference formula to find a solution. This shows that HM has not understood the material before, so the classified objects are less appropriate. Whereas in question number 2, HM can classify objects according to certain properties, but there is a slight error, namely HM rating $\sec^0 x$ is changed to $\tan^2 x + 1$

Ability to select, use, and utilize specific procedures or operations

Based on question number 1, for indicators of the ability to choose, use, and utilize certain procedures or operations HM answers correctly but makes many mistakes, namely for solving the trigonometric equation $\sin x = 0$ has two solutions and the trigonometric equation $\cos 2x = 0$ has two solutions, but in work, it is not visible. In question number 2, for indicators of the ability to choose, use, and utilize certain procedures or operations HM answers correctly but there are many mistakes, namely not doing the reasoning to change in the form of squares but directly working on and not writing down how the steps get the set of solutions.

Ability to apply problem-solving concepts

Based on question number 1, for indicators applying the concept of problem-solving HM applies the concept of problem-solving, but it is not appropriate. HM solves the problem by applying the angular difference formula and then looking for a solution in the form of a trigonometric equation. In addition, in question number 2, for this indicator, HM applies the concept of problem-solving, but it is not appropriate that the solution does not use trigonometric equations for the size of radian angles, but HM directly answers in degrees and converts from degrees to radians

Based on the discussion above, subjects with learning independence in the high category have a better understanding of mathematical concepts than subjects who have to learn independence in the low category. Research conducted by Prastiyo showed that subjects with high category learning independence have a high ability to understand mathematical concepts, subjects with medium category learning independence can understand mathematical concepts that are also, and subjects with low category learning independence also have a low ability to understand mathematical concepts (Prastiyo et al., 2021). However, based on the results of the study, subjects with medium learning independence have a low understanding of mathematical concepts due to the subject's ability to apply problem-solving concepts that are not good. This condition is also to the results of research that shows that student learning independence is good, but the level of understanding of students' concepts towards the material is still very low (Anggraeni, 2016). In addition, other studies show that high learning independence does not necessarily have a high understanding of mathematical concepts. The thing that causes is the difficulty of linking between concepts and the ability to understand problems (Izzati, Sholikhakh, & Amaliyakh, 2021).

CONCLUSION

Based on the results of the data analysis carried out, it can be concluded that subjects with learning independence in the high category can understand mathematical concepts by 88% and are in the high category, while subjects who have a moderate level of learning independence can understand mathematical concepts of 42% and are in a low category.

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