# Application of Number Theory in the Calculation of Java Calendar 

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#### Abstract

This paper is a study of the theory and application of number theory. The purpose of this study is to examine a concept and then can be applied and implemented in mathematical computation so as to facilitate and support aspects of life outside mathematics. Application of number theory is widely used in everyday life one of them is to determine of the market day in the Java calendar. The concept of number theory used in the calculation of Java calendar is the concept of modulo 7 and modulo 5. The concept modulo 7 is used to see the day in the Masehi Calendar whereas modulo 5 is used to see the market day in the Javanese Calendar.


## 1. Introduction

The mathworld.wolfram.com says that number theory is a vast and fascinating field of mathematics, sometimes called "higher arithmetic," consisting of the study of the properties of whole numbers. The great difficulty in proving relatively simple results in number theory prompted no less an authority than Gauss to remark that "it is just this which gives the higher arithmetic that magical charm which has made it the favourite science of the greatest mathematicians, not to mention its inexhaustible wealth, where in it so greatly surpasses other parts of mathematics." Gauss, often known as the "prince of mathematics," called mathematics the "queen of the sciences" and considered number theory the "queen of mathematics" [1-2].

Theory of Numbers has become the basis of the development of several branches of mathematics such as cryptography (secret writing/password) and computer science as one of development in applied mathematics. Primes and prime factorization are especially important in number theory, as are a number of functions such as the divisor function, Riemann zeta function, and totient function. The modulo system is an important part of the Theory of Numbers [3].

One of the uses of a very interesting modulo system is to determine the day and the market, both past and future days and markets. Condition is the date, month and year to be sought days and the market is known with certainty.

Often we experience events to determine the day and market of a date that we consider so historic to us we can't successfully remember correctly. Want to see the calendar has been torn or even no longer exists. Due to the difficulty of finding calendars of past years to determine the day and market of important dates is a matter to be solved or the answers are general and mathematical. Therefore, the following will present some easy and simple ways for that purpose.

The way to be presented below is not only used to remember past days and markets, but also to determine the day and market to become. Do not disassemble the document or compile a new calendar, as long as it is known with certainty the date, month, and year in question and think for a moment.

## 2. Masehi Calendar

Determination day or week in calendar can use modulo 7 because days in a week there are 7 days. Days of the week can be recorded with the numbers $0,1,2,3,4,5,6$ with the rules:

| Sunday | $=0$, | Wednesday | $=3$, | Saturday $=6$ |
| :--- | :--- | :--- | :--- | :--- |
| Monday | $=1$, | Thursday | $=4$, |  |
| Tuesday | $=2$, | Friday | $=5$, |  |

Julius Caesar changed the Egyptian calendar numbering 365 days in a year to 365 1/4 days in a year called the Julian calendar with leap year leap every four years. The last calculation shows the actual length of each year is about 365.2422 days. Several centuries after the addition of 0.0078 days per year are not appropriate, so in 1582 an estimated 10 days was added from the previous year that was skipped [4].

In 1582, Paus Gregory made a new calendar. First, 10 days were added, so the date of October $5^{\text {th }}, 1582$ to October $15^{\text {th }}, 1582$ (October 6-14 ${ }^{\text {th }}$ is bypassed). It has been established that the leap year will be appropriately placed in a year that can divisible 4, except in the 100 divisible centenary year, but it would be possible if exactly divided by 400 . For example, 1700, 1800, 1900, and 2100 are not leap years, but 1600 and 2000 are leap years. With this rule, the average length of the calendar a year is to 365.2425 days, close to the actual year of 365.2422 days. The error estimate is 0.0003 days per year that leaving 3 days per 10,000 years.

Perpetual calendars are used to specify the day or week of the date on the Gregorian calendar. Because of the excess days in leap year in February then the numbering in the month starting in March and assume that January and February is part of the end of the year. For example February 2000 was considered as the twelfth month of 1999 , and May 2000 was considered as the third month in 2000. With the following numbering provisions:

- $k=$ days in month
- $m=$ Month

| January $=11$ | May $=3$ | September $=7$ |
| :--- | :--- | :--- |
| February =12 | June $=4$ | October $=8$ |
| March $=1$ | July $=5$ | November $=9$ |
| April $=2$ | August $=6$ | December $=10$ |

- $\mathrm{N}=$ year, $N$ is the current year except in January or February, then N is the year before, and $\mathrm{N}=100 \mathrm{C}+\mathrm{Y}$
- $C=$ century
- $Y=$ description of the year in the century

For example, on April $3^{\text {rd }}, 1951, \mathrm{k}=3, \mathrm{~m}=2, \mathrm{~N}=1951, \mathrm{C}=19$, and $\mathrm{Y}=51$. Note for February $28^{\text {th }}, 1951, \mathrm{k}=28, \mathrm{~m}=12, \mathrm{~N}=1950, \mathrm{C}=19$, and $\mathrm{Y}=50$, since in the calculation is considered February is the twelfth month of the previous year.

March $1^{\text {st }}$, of each year is used as the basis for calculation. Let $d_{N}$ represent the day of the week from March $1^{\text {st }}$ of the year N. Beginning from the year 1600 and calculating the day of the week of March $1^{\text {st }}$ falling every year. Note that between March $1^{\text {st }}$ of $\mathrm{N}-1$ and March $1^{\text {st }} \mathrm{N}$ years, if N is not a leap year, there will be 365 days, and since $365 \equiv 1(\bmod 7), \mathrm{d}_{\mathrm{N}} \equiv \mathrm{d}_{\mathrm{N}-1}+1(\bmod 7)$, whereas if N is a leap year there is an additional day between the first March sequence, so
$\mathrm{d}_{\mathrm{N}} \equiv \mathrm{d}_{\mathrm{N}-1}+2(\bmod 7)$

The preceding determinants of how many leap years occurred between 1600 and N (excluding 1600 but including N ), are called x . To take into account x , the first note that by dividing the algorithm there is $[(\mathrm{N}-1600) / 4]$ years can be divided by 4 between 1600 and N , there [ $[\mathrm{N}-$ $1600] / 100$ ] years can be divided by 100 between 1600 and N , and there are [ $(\mathrm{N}-1600) / 400$ ] years can be divided by 400 between 1600 and N. Therefore,

$$
\begin{aligned}
x & =[(\mathrm{N}-1600) / 4]-[(\mathrm{N}-1600) / 100]+[(\mathrm{N}-1600) / 400] \\
& =[\mathrm{N} / 4]-400-[\mathrm{N} / 100]+16+[\mathrm{N} / 400]-4 \\
& =[\mathrm{N} / 4]-[\mathrm{N} / 100]+[\mathrm{N} / 400]-388
\end{aligned}
$$

Substitution $\mathrm{N}=100 \mathrm{C}+\mathrm{Y}$ :

$$
\begin{aligned}
\mathrm{x} & =[25 \mathrm{C}+(\mathrm{Y} / 4)]-[\mathrm{C}+(\mathrm{Y} / 100)]+[(\mathrm{C} / 4)+(\mathrm{Y} / 400)]-388 \\
& =25 \mathrm{C}+[\mathrm{Y} / 4]-\mathrm{C}+[\mathrm{C} / 4]-388 \\
& \equiv 3 \mathrm{C}+[\mathrm{C} / 4]+[\mathrm{Y} / 4]-3(\bmod 7)
\end{aligned}
$$

Calculates $d_{N}$ of $d_{1600}$ by substituting $d_{1600}$ with one day for each year bypassed, plus an additional annually of leap year between 1600 and N. Obtained formula:

$$
\begin{aligned}
\mathrm{d}_{\mathrm{N}} & \equiv \mathrm{~d}_{1600}+\mathrm{N}-1600+\mathrm{x} \\
& =\mathrm{d}_{1600}+100 \mathrm{C}+\mathrm{Y}-1600+3 \mathrm{C}+[\mathrm{C} / 4]+[\mathrm{Y} / 4]-3(\bmod 7) \\
& \text { The simplification becomes } \\
\mathrm{d}_{\mathrm{N}} & \equiv \mathrm{~d}_{1600}-2 \mathrm{C}+\mathrm{Y}+[\mathrm{C} / 4]+[\mathrm{Y} / 4](\bmod 7)
\end{aligned}
$$

A formula has been found that links the day of the week to March $1^{\text {st }}$ of the year to the day of the week to March $1^{\text {st }}, 1600$. Using the fact that March $1^{\text {st }}, 1982$ is Monday to find the day of the week for March $1^{\text {st }}, 1600$.
In 1982, $\mathrm{N}=1982$, then $\mathrm{C}=19, \mathrm{Y}=82$, and $\mathrm{d}_{1982}=1$, so

$$
\mathrm{d}_{\mathrm{N}} \equiv \mathrm{~d}_{1600}-2 \mathrm{C}+\mathrm{Y}+[\mathrm{C} / 4]+[\mathrm{Y} / 4](\bmod 7)
$$

$1 \equiv \mathrm{~d}_{1600}-38+82+[19 / 4]+[82 / 4] \equiv \mathrm{d}_{1600}-2(\bmod 7)$ so that
$d_{1600}=3$, so March 1st 1600 is Wednesday. By entering the value of $d_{1600}$, the formula for $d_{N}$ becomes

$$
d_{N} \equiv 3-2 C+Y+[C / 4]+[Y / 4](\bmod 7)
$$

The formula is used to calculate the day of the week on the first day of each month of the year N. Must use the number of the earliest days of the month from a particular month is shift from the beginning of the month from the previous month. The month with 30 days shifted the beginning of next month as much as 2 days due $30 \equiv 2(\bmod 7)$. The month with 31 days shifts the beginning of next month as much as 2 days due $31 \equiv 3(\bmod 7)$. The addition of each month can be written as follows:

| March $1^{\text {st }}$ to April $1^{\text {st }}$ | : 3 days |
| :---: | :---: |
| April $1^{\text {st }}$ to May $1^{\text {st }}$ | : 2 days |
| May ${ }^{\text {st }}$ to June $1^{\text {st }}$ | : 3 days |
| June $1^{\text {st }}$ to July $1^{\text {st }}$ | : 2 days |
| July $1^{\text {st }}$ to August $1^{\text {st }}$ | : 3 days |
| August $1^{\text {st }}$ to September $1^{\text {st }}$ | : 3 days |
| September $1^{\text {st }}$ to October $1^{\text {st }}$ | : 2 days |
| October $1^{\text {st }}$ to November $1^{\text {st }}$ | : 3 days |
| November $1^{\text {st }}$ to December $1^{\text {st }}$ | : 2 days |
| December $1^{\text {st }}$ to January $1^{\text {st }}$ | : 3 days |
| January $1^{\text {st }}$ to February $1^{\text {st }}$ | : 3 days |

The total number of additional days a year is 29 days, so for each additional average of 2.6 days. The function [ $2.6 \mathrm{~m}-0.2$ ] -2 has an exact addition with m of 2 to 12 , and zero when $\mathrm{m}=1$ (this formula was found by Christian Zeller by trial and error). Thus, the day of the week at the beginning of the month of the month $m$ of year $N$ gives the non-negative residual of $d_{N}+[2.6 \mathrm{~m}-0.2]-2$ modulo 7. Determination of W days in a week from day k of month m year N , add $\mathrm{k}-1$ to the formula which has been found for the day of the week on the first day of the same month, obtained the formula $\mathrm{W} \equiv \mathrm{k}+[2.6 \mathrm{~m}-0.2]-2 \mathrm{C}+\mathrm{Y}+[\mathrm{C} / 4]+[\mathrm{Y} / 4](\bmod 7)$ The formula can be used to find the day of the week of a date on a year in the Gregorian calendar. Example Determination of the day on August 17, 1945, $\mathrm{k}=17, \mathrm{~m}=6, \mathrm{C}=19 \mathrm{Y}=45$. So $\mathrm{W} \equiv \mathrm{k}+[2.6 \mathrm{~m}-0.2]-2 \mathrm{C}+\mathrm{Y}+[\mathrm{C} / 4]+[\mathrm{Y} / 4]$ $(\bmod 7) \equiv 17+15-38+45+4+11 \equiv 54 \equiv 5(\bmod 7)$.
So August $17^{\text {th }}, 1945$ is Friday.

## 3. Javanese Calendar

The Javanese Calendar is the dating system used by the Sultanate of Mataram and its various ruptured kingdoms and its influence. The calendar has the distinctive features of combining the Islamic calendar system, the Hindu Calendar system, and the little calendar of Julian that is part of Western culture [5].

The Javanese calendar system uses two day cycles: weekly cycles consisting of seven days (Sunday to Saturday) and a five week cycle of pancawara weekends. In 1625 AD (1547 Saka), Sultan Agung of Mataram tried hard to instill Islam in Java. One of his efforts is to issue a decree that replaces the Sun-based Saka calendar with a lunar or lunar calendar system (based on the rotation of the moon). Uniquely, the number of Saka years remains used and continued, not using the calculations of the Hijri year (at that time 1035 H ). This is done for the sake of sustainability, so that the year that is the year 1547 Saka forwarded to the year 1547 Java.

In the Java calendar we recognize the term of the day of the market is a day to hold a market in a particular region. The market day in the Java calendar is Pon, Wage, Kliwon, Paing and Legi. To determination day or week in Java calendar can use modulo 5 because the days in a Java week are 5 days. Days of the week can be recorded with the numbers $0.1,2,3,4$ with the rule: Pon $=0$, Legi $=3$, Wage $=1$, Paing $=4$, Kliwon $=2$, To search market day in Java calendar, we need table the big market that is used as a summation aid with the date we will look for in the Masehi calendar [6]. The big calendar table for Java calendar calculation is as follows: Example to search the market day of Java for the day of proclamation Indonesia (August 17 ${ }^{\text {th }}$ 1945), we look for the year 1945 August in the big market table. It is obtained the number 1 . Then the number 1 we sum up with the date that we will search so obtained $1+17=18$. To find the market day in the Java calendar, we calculate modulo 5 of $18.18 \equiv 3(\bmod 5)$ so we can know that the market day on August $17^{\text {th }}, 1945$ is Legi. So the date of August $17^{\text {th }}, 1945$ is on Legi Friday.

## 4. References

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Appendix
Table of Big Javanese Market Days

| Year |  |  |  |  |  |  | Jan | Feb | Mar | April | May | Jun | Jul | Agust | Sept | Okt | Nov | Dec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1880 | 1904 | 1924 | 1944 | 1964 | 1984 | 2004 | 3 | 4 | 3 | 4 | 4 | 0 | 0 | 1 | 2 | 2 | 3 | 3 |
| 1881 | 1905 | 1925 | 1945 | 1965 | 1985 | 2005 | 4 | 0 | 3 | 4 | 4 | 0 | 0 | 1 | 2 | 2 | 3 | 3 |
| 1882 | 1906 | 1926 | 1946 | 1966 | 1986 | 2006 | 4 | 0 | 3 | 4 | 4 | 0 | 0 | 1 | 2 | 2 | 3 | 3 |
| 1883 | 1907 | 1927 | 1947 | 1967 | 1987 | 2007 | 4 | 0 | 3 | 4 | 4 | 0 | 0 | 1 | 2 | 2 | 3 | 3 |
| 1884 | 1908 | 1928 | 1948 | 1968 | 1988 | 2008 | 4 | 0 | 4 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 |
| 1885 | 1909 | 1929 | 1949 | 1969 | 1989 | 2009 | 0 | 1 | 4 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 |
| 1886 | 1910 | 1930 | 1950 | 1970 | 1990 | 2010 | 0 | 1 | 4 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 |
| 1887 | 1911 | 1931 | 1951 | 1971 | 1991 | 2011 | 0 | 1 | 4 | 0 | 0 | 1 | 1 | 2 | 3 | 3 | 4 | 4 |
| 1888 | 1912 | 1932 | 1952 | 1972 | 1992 | 2012 | 0 | 1 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 0 | 0 |
| 1889 | 1913 | 1933 | 1953 | 1973 | 1993 | 2013 | 1 | 2 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 0 | 0 |
| 1890 | 1914 | 1934 | 1954 | 1974 | 1994 | 2014 | 1 | 2 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 0 | 0 |
| 1891 | 1915 | 1935 | 1955 | 1975 | 1995 | 2015 | 1 | 2 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 0 | 0 |
| 1892 | 1916 | 1936 | 1956 | 1976 | 1996 | 2016 | 1 | 2 | 1 | 2 | 2 | 3 | 3 | 4 | 0 | 0 | 1 | 1 |
| 1893 | 1917 | 1937 | 1957 | 1977 | 1997 | 2017 | 2 | 3 | 1 | 2 | 2 | 3 | 3 | 4 | 0 | 0 | 1 | 1 |
| 1894 | 1918 | 1938 | 1958 | 1978 | 1998 | 2018 | 2 | 3 | 1 | 2 | 2 | 3 | 3 | 4 | 0 | 0 | 1 | 1 |
| 1895 | 1919 | 1939 | 1959 | 1979 | 1999 | 2019 | 2 | 3 | 1 | 2 | 2 | 3 | 3 | 4 | 0 | 0 | 1 | 1 |
| 1900 | 1920 | 1940 | 1960 | 1980 | 2000 | 2020 | 2 | 3 | 2 | 3 | 3 | 4 | 4 | 0 | 1 | 1 | 2 | 2 |
| 1901 | 1921 | 1941 | 1961 | 1981 | 2001 | 2021 | 3 | 4 | 2 | 3 | 3 | 4 | 4 | 0 | 1 | 1 | 2 | 2 |
| 1902 | 1922 | 1942 | 1962 | 1982 | 2002 | 2022 | 3 | 4 | 2 | 3 | 3 | 4 | 4 | 0 | 1 | 1 | 2 | 2 |
| 1903 | 1923 | 1943 | 1963 | 1983 | 2003 | 2023 | 3 | 4 | 2 | 3 | 3 | 4 | 4 | 0 | 1 | 1 | 2 | 2 |

