

Thinking Obstacles with Scaffolding in Solving The Limit of Sequence Problems in Mathematics' Students of Hasyim Asy'ari University

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Abstract. Real analyze is one of the lectures which are very important in exercise the structured and rational deductive thinking. However, most students of mathematics is still having a hard time to understand it, especially on the matter sequence of real numbers. The difficulties indicate the existence of obstacles on students in problem solving on a sequence of real numbers. The aim of this research is describe obstacles thinking of students of mathematics education department in problem solving on a sequence of real numbers. The approach was conducted in this research is descriptive qualitative. The subject in this study were 6 students with high, middle and low ability. This research focused on the analysis of obstacles thinking that facing on students of mathematics in solve limit sequence problems. The problem solving in this research based on problem solving of Polya, are understanding the problems, planning the solve, conducting the solve, and reviewing the procedure and result. The students' obstacles consisting of difficulties to think that due to the lack in ability of students' analogies, the lack in ability of students' connection in mathematics, and the lack of initial knowledge owned by the students. To address thinking obstacles which facing by the students in solving sequence of real numbers such that is give the scaffolding by lecturer. The scaffolding *there are*: (a) Level 1 (environmental provisions), (b). Level 2 (explaining, reviewing and restructuring), and (c). Level 3(developing conceptual thinking).

1. Introduction

One of the branches of science that has an important role in the development of a generation with high reason is mathematics. This can be seen from the many roles of mathematics in science and the world of work [1]. Many problems in daily activity can be modeled in mathematical form [2]. Therefore, mathematics lesson are given from an early age to the lecture bench level.

One of the most important mathematics lessons in terms of formal thinking ability is the real analysis [3]. The basic real analysis is one of the compulsory subjects of the mathematics course at Hasyim Asy'ari University with 3 credits. Wuryanto [4] said that the real analysis depends on the discernment of the analysis by using a logical flow that is valid. Real analysis aims to train students to think structured and rational deductive. This can be seen from the many problems about mathematical proofing. Although real analysis is a very important lesson, most mathematics students still have trouble understanding it. Real analysis is considered difficult by students because it has abstract material [5]. This is as stated by Harini [6] that the real analysis is still impressed using conventional learning so that it becomes a scourge for students in math majors of various universities in Indonesia.

One of the materials that are considered difficult for students in the real analysis is the sequence of real numbers [7]. One of the difficulties in studying material limits is regarding the definition of limit [8]. Duru [9] conducted research on the teachers and the result was that many teachers still do not understand correctly about the concept of limit. Nevertheless, there is still no research that seeks to find the cause of the difficulties experienced by students in studying real analysis. Research on the thinking obstacle of students in solving the problem of sequence of real numbers still has not been done. The difficulties experienced by students in the material of the sequence of real numbers indicate the thinking obstacle of students in solving the problems in the sequence of real numbers.

Many studies suggest that environment affects learning outcomes and the ability to understand mathematics. Nurhayati [10] states that the non-social environment is a physical environment or a means used that have an influence on educational activities. Utami [11] also conveyed that there are environmental influences on mathematics learning outcomes. The influence of environment on learning outcome or mathematics learning achievement was also presented in Meifiani [12]. This study will also discuss about student's barriers to thinking in different neighborhoods.

A possible way for lecturers to overcome the thinking obstacle is to provide scaffolding. Amintoko [13] states that scaffolding is an aid given by other people to the child in order to carry out his own work. Scaffolding can be done in 3 levels: Scaffolding level 1 (environmental provisions), Scaffolding level 2 (explaining, reviewing, and restructuring), and Scaffolding level 3 (developing conceptual thinking) [14].

2. Research Methods

2.1 Approach and Type of Research

Research on the thinking obstacle of mathematics students is a descriptive-qualitative research. This study is intended to obtain a description of the thinking obstacle experienced by students in solving the problem of the sequence of real numbers and the scaffolding by the lecturers to overcome them. The subject of this study is a mathematics student who has taken the subject of real analysis of 6 students of mathematics semester VI. Of the students of mathematics semester VI there are students with low ability, high, and middle so that expected data in the form of thinking obstacle can represent the actual conditions in the field. In addition, subject selection also represents the conditions of two different environments of student residence. The determination of student ability is based on preliminary test, semester midterm score, and discussion with some lecturers who also teach in that class. Methods of data collection in this study using the method of disclosure of test results and interviews. Interviews conducted have the purpose to reveal the barriers to student thinking and provide scaffolding.

2.2 Research Instrumen

In addition to the researchers as the main instrument, there are several other instruments used in this study. Scaffolding guide sheets and test questions to uncover student obstacle thinking were also instrumental in this study. As for the test questions given to reveal the students obstcale thinking are as follows:

Solve the Folowing Question with 30 minutes:

1. $M = (X_n)$ is defined
 $X_1 = 1, X_2 = 1, X_{n+1} = X_{n-1} + X_n$ ($n \geq 2$).
 - a. Is M defined as sequence on Real Numbers?
 - b. Can members of M be paired exactly 1 pair with natural Numbers? Explain your answer!
2. Determine whether the sequences are convergent or not. If it convergent then determine which convergence of the sequence.

- a. $(A_n) = \left(\frac{n^2}{2n^2+1}\right)$
 - b. $(B_n) = \left(\frac{(-1)^n}{n+2}\right)$
 - c. $(C_n) = (1 + (-1)^n)$
 - d. $(D_n) = \left(\frac{n^2}{n!}\right)$
 - e. $(E_n) = \left(2 + \frac{1}{n}\right)^2$
3. Based on the definition of convergence, find the magnitude of the original number $K(\varepsilon)$ so that for all natural numbers $n \geq K(\varepsilon)$ obtain $\frac{1}{\sqrt{n+7}} < \varepsilon$ at $\lim \left(\frac{1}{\sqrt{n+7}}\right) = 0$. Explain your answer!
4. Prove that $(Y_n) = (-1)^n$ is not convergent. Explain your answer!
5. Prove the limit of the following sequence with the definition of epsilon-delta and explain using the graph:
- a. $\lim \left(\frac{n}{n^2+1}\right) = 0$
 - b. $\lim \left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}$

3. Result and Discussion

Based on the test results reveal the obstacle thinking experienced by students. Obstacle thinking experienced by students are as follows:

Table 1. Students Obstacle Thinking

No	Subjek	Environment	Obstacle Thinking on the Question Number
1.	S1	Home	1. – Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students’ analogies
			2. – Obstacle thinking due to the lack in ability of students’ connection in mathematics - Obstacle thinking due to the lack in ability of students’ analogies
			3. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students’ connection in mathematics
			4. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students’ connection in mathematics - Obstacle thinking due to the lack in ability of students’ analogies
			5. - Obstacle thinking due to the lack of initial knowledge owned by the students

No	Subjek	Environment	Obstacle Thinking on the Question Number
			<ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies
2.	S2	Pesantren	<ol style="list-style-type: none"> 1. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' analogies 2. - Obstacle thinking due to the lack in ability of students' connection in mathematics <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' analogies 3. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics 4. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies 5. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies
3.	S3	Home	<ol style="list-style-type: none"> 1. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' analogies 2. - Obstacle thinking due to the lack in ability of students' connection in mathematics <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' analogies 3. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics 4. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies 5. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of

No	Subjek	Environment	Obstacle Thinking on the Question Number
			students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies
4.	S4	Pesantren	<p>1. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' analogies</p> <p>2. - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p> <p>3. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics</p> <p>4. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p> <p>5. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p>
5.	S5	Home	<p>1. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p> <p>2. - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p> <p>3. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics</p> <p>4. - Obstacle thinking due to the lack of initial knowledge owned by the students - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies</p> <p>5. - Obstacle thinking due to the lack of initial knowledge owned by the students</p>

No	Subjek	Environment	Obstacle Thinking on the Question Number
			<ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies
6.	S6	Pesantren	<ol style="list-style-type: none"> 1. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies 2. - Obstacle thinking due to the lack in ability of students' connection in mathematics <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' analogies 3. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies 4. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies 5. - Obstacle thinking due to the lack of initial knowledge owned by the students <ul style="list-style-type: none"> - Obstacle thinking due to the lack in ability of students' connection in mathematics - Obstacle thinking due to the lack in ability of students' analogies

From the table above can be seen that students who are in the home environment and in the pesantren environment have some similarities in the obstacle of thinking. Nevertheless there are some differences from each subject about the obstacle thinking.

On the number 1 problem, the subject with high mathematical ability that is S1 and S2. For subject S1 who live in the home environment and also S2 who live in pesantren environment have similarities that is The students' obstacles consisting of difficulties to think that due to the lack in ability of students' analogies and the lack of initial knowledge owned by the students. S1 and S2 have been able to understand the problem well and have been able to connection information on the problem with the question. However, S1 and S2 do not write the information on the problem completely, do not write down what is known and the question on the problem, and do not describe in the form of diagrams between the natural numbers and members of M. The difference between S1 and S2 in answer about number 1 is S2 has responded by signing up so it's easier and more helpful towards the desired answer. Even so S1 also has the idea of registering but not writing that way.

S1 and S2 also forget about the concept of function as well as bijective so the resume is wrong answer. Scaffolding given in question number 1a is scaffolding at reviewing and restructuring level. While on the question number 1b given Scaffolding start at the level of environmental

provisions, explaining, reviewing and restructuring. The number of Scaffolding on the 1b problem is due to lack of early understanding of students.

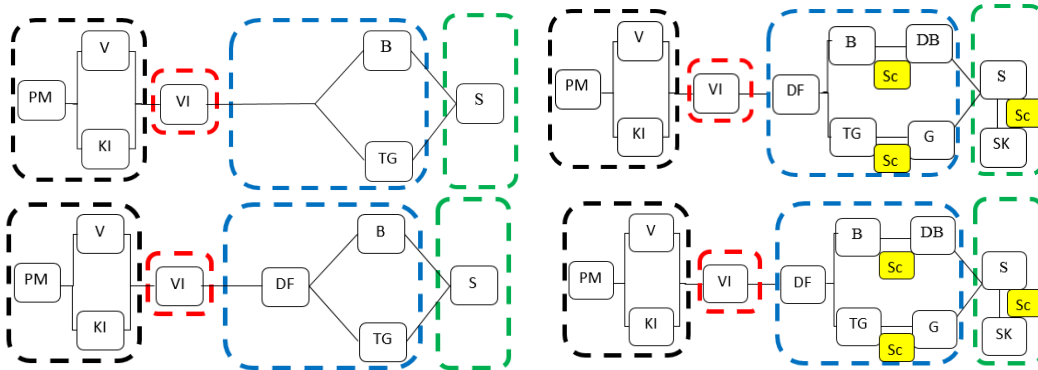


Figure 1. Structure of S1 and S2 thinking on number 1 before and after being given scaffolding

In the case of number 2 problem, S1 and S2 have similarities that is obstacles consisting of difficulties to think that due to the lack in ability of students' analogies and the lack in ability of students' connection in mathematics. S1 and S2 have been able to understand the problem well and have been able to connection information on the problem with the question. However, S1 and S2 do not write information on the problem completely, do not write down what is known and the question on the problem. The difference between S1 and S2 on the answer about the number 2 is S1 has answered in two ways that is substitution and also by limit while S2 only finish by substitution.

S1 and S2 also difficulties in analogizing convergent sequence and also divergent sequences. This can be seen in the error made by S1 in concluding the answer of the sequences. Scaffolding given in question number 2 is starting at the level of environmental provisions, explaining, reviewing and restructuring. This is necessary because there is a problem that is still wrong when resolved by substitution.

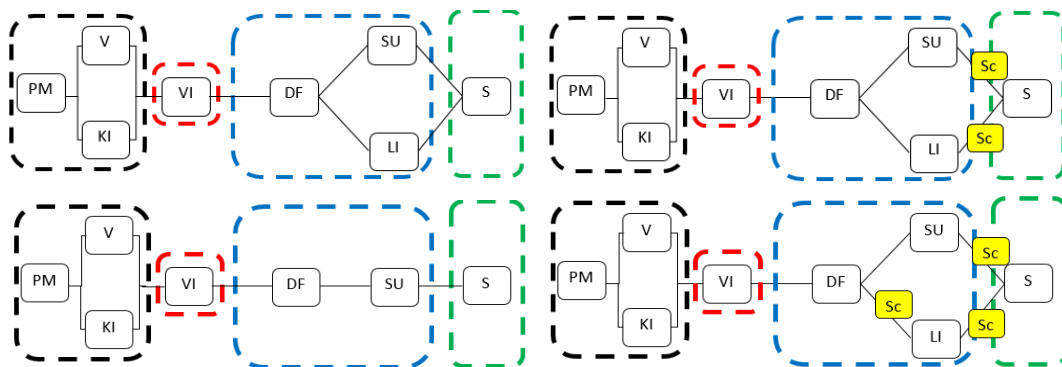


Figure 2. Structure of S1 and S2 thinking on number 2 before and after being given scaffolding

On the question number 3, S1 and S2 have been able to understand the problem well and was able to find ideas to solving the problem. However S1 and S2 are not able to continue the idea and make mistakes in determining the value of K. Differences between S1 and S2 on the answer from number 3 problem is S1 more thorough and meticulous in writing the problem and use the definition of convergent while S2 less accurate so wrong in writing the problem. Scaffolding given in Problem 2 is Scaffolding starting at the level of environmental provisions, explaining, reviewing and

restructuring. This is necessary because from the beginning the students have difficulty in connecting the information and the concept used is also wrong.

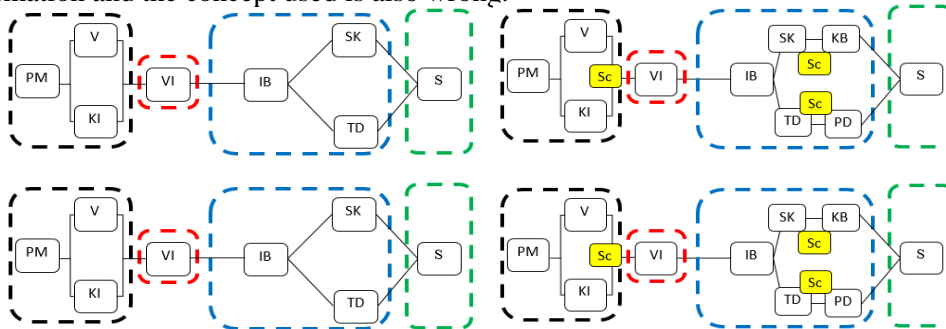


Figure 3. Structure of S1 and S2 thinking on number 3 before and after being given scaffolding

In the case of number 4 problem, S1 and S2 are equally no answer. S1 and S2 can not visualize and can not retrieve information on the problem. S1 and S2 also not find the idea, not write information on the problem, and not write down what is known and the question on the problem. The difference between S1 and S2 in answer about number 4 is that S1 knows more about the definition of convergent and divergent.

S1 and S2 experience obstacles consisting of difficulties to think that due to the lack in ability of students' analogies, the lack in ability of students' connection in mathematics, and the lack of initial knowledge owned by the students. Scaffolding given in question number 4 also starts from the level of environmental provisions, explaining, reviewing and restructuring. Many scaffoldings given due to S1 and S2 experiencing barriers to thinking from the beginning to understand the problem until the solution.

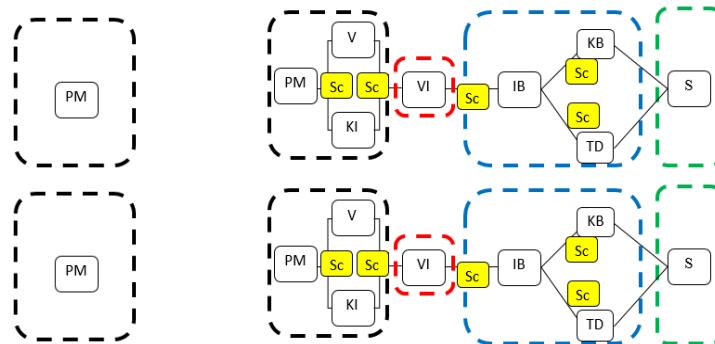


Figure 4. Structure of S1 and S2 thinking on number 4 before and after being given scaffolding

On the question number 5, S1 has understood the problem by using the concept of epsilon delta as in problem number 3. In addition S1 also describes the graphics even though the graphics are made incorrect. However, S1 is not able to continue the idea so that the concept used is also still wrong. While S2 has not been able to understand the problem and have not been able to visualize the problem correctly. S2 only gives answers in the form of graphic images without proving by definition epsilon-delta. The graphic image given is still incorrect because does not understand correctly the definition of convergence. S1 and S2 are both experience obstacles consisting of difficulties to think that due to the lack in ability of students' analogies, the lack in ability of students' connection in mathematics, and the lack of initial knowledge owned by the students. Scaffolding given in Problem 5 for S2 starts from the level of environmental provisions, explaining, reviewing and restructuring.

While the scaffolding given on the problem number 5 for S1 starts from the level explaining, reviewing and restructuring.

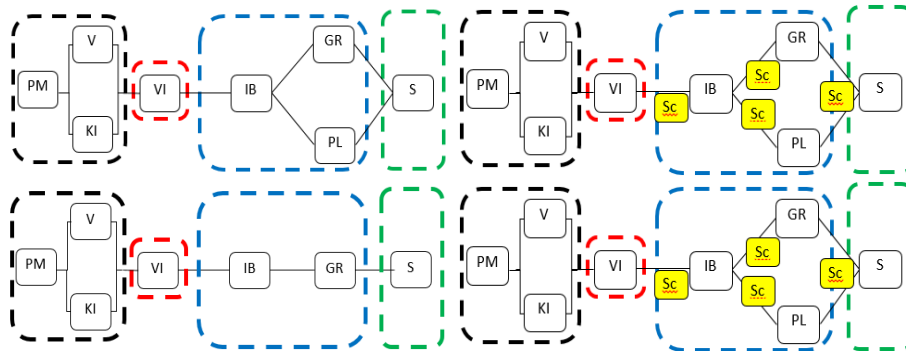


Figure 5. Structure of S1 and S2 thinking on number 5 before and after being given scaffolding

From the 5 issues above it can be seen that there are some similarities of constraints experienced by S1 and S2 on tests reveal of thinking obstacle and also understanding when given scaffolding between students living at the home environment and students living at the pesantren environment. This is because the number of student activities in pesantren. But the difference is not so significant. This is also in accordance with previous research conducted by Utami [11] and Meifiani [12] on environmental influences on mathematics learning outcomes. The impact given is not too significant as previous research conducted by Meifiani [12] and Nurhayati [10].

Table 2. Code and explanation of the meaning of the code in the image of the structure of thinking

V	can visualize the problem
KI	can connect the information on the problem
VI	connection information with the visualization obtained
TV	can not visualize the problem
DF	determine the answer by registering
B	answered without knowing the definition of the sequence
DB	answer by knowing the definition of sequence
TG	answered without a diagram
G	Answered with diagram
S	Solutions produced by students
SK	solutions produced by students accompanied by information
Sc	giving scaffolding
SU	solution with substitution
LI	solution with limit and squeeze rules
IB	the first idea used is correct
KS	the concept used is wrong
KB	the concept used is correct
TD	without knowing the definition of convergent and divergent
PD	understand definition
GR	can draw graph
PL	can finish with epsilon delta
IS	wrong idea first

PM	Can understand the problem
Information:	
— — — —	: <u>understanding the problems</u>
— — — —	: Planning the Solve
— — — —	: <u>reviewing the procedure and result</u>
— — — —	: Conducting the Solve

4. Conclusion

Based on the result of research, it is found that the obstacle of students' thinking in solving the problem of limit sequence for the students who live in the home environment and the students in the pesantren environment are 3, namely: 1. students obstacles consisting of difficulties to think that due to the lack in ability of students' analogies, 2. the lack in ability of students' connection in mathematics, and 3. the lack of initial knowledge owned by the students. Although in terms of barriers to thinking there are some similarities but there are some differences that is in terms of doing tests expressing obstacle of thinking and in understanding the problem when given scaffolding. The lecturers do to overcome these obstacles is to provide scaffolding. At the environmental provisions stage for example by making an arrow diagram when installing exactly one number. In the explaining stage include asking students to explain about the sequence of convergent and non convergent numbers. In reviewing stage such as reflection on the answers and improve the results of his work. At the restructuring stage is by giving directions to reply back correctly. At the stage of scaffolding developing conceptual thinking in the form of looking for alternative ways for example by using limit ranks.

This research is a qualitative descriptive research so that it can be given the suggestion that more and more subject taking will be more and more good data obtained.

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