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Arithmetic thinking: Reflect on the order of operation

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Abstract. Arithmetic operations became a basic understanding before acquiring algebraic thinking. Properties of the operation including its order has important role in solving algebraic expression problem later on. However, research in Indonesia rarely explored prospective teachers' understanding on it. There was no popular mnemonic used as the common rule for ordering arithmetic operation. This study was a descriptive qualitative research. It involved 30 prospective teachers from mathematics and elementary teacher education program in Surabaya. Their understanding was described based on their responses for two types of problem. It was solving four integer calculations. Their paperwork provided reflection on how the operation was carried out. It was also supported by a questionnaire that measures their comprehension on the order. Typical errors were made when choosing which operation came first. The problems were between addition (A) or subtraction (S) and multiplication (M) or division (D). No confusion other than miscalculation for the case picking AS or MD.

1. Introduction

Arithmetic operation is one of requirement for algebraic thinking. Arithmetic operation on integers serves as a requirement to learn algebra. Simple operation included addition, subtraction, and multiplication on algebraic expression involved arithmetic operation. It was the introduction topic of algebraic expression [1]. Understanding the order of arithmetic operation for integers would support student to learn algebraic expression.

One important property that signifies arithmetic is the order of the operation. The more complex concept of it was needed when students start to learn algebra course [2]. Mathematics classroom across the world contained the order of operation as part of the arithmetic course. Some research related to how this topic comprehended by students was carried out. These study mostly concern about how the mnemonic used as the rule for ordering the operation [2–5].

The most popular acronym for the order of operation is PEMDAS. It stands for Parenthesis, Multiplication, Division, Addition, and Subtraction. In Canada, this mnemonic spells differently by replacing Parenthesis with Brackets [5]. It also changes the order between multiplication and division. It is called BEDMAS. However, PEMDAS lead pre-service elementary school teachers to think that it was the strict order of arithmetic operation [6].

In Indonesia, some people know Kabataku (Kali, Bagi, Tambah, Kurang). It is translated into (Times, divide, plus, minus). Javanese people have special acronym for arithmetic operation called PIPOLONDO (*Ping, Poro, Lan, Sudo)*. It translated as multiply, divide, add, and subtract. In addition, there was no parenthesis or brackets included in the acronym. It was also rarely discussed purposively in the classroom to teach the order.

Related to these acronyms and its use, this specific part of arithmetic topic in Indonesia was not explored in a research. Apart from school students' understanding, it was more interesting how the prospective teacher in Indonesia understand this concept. There was a research that involved prospective elementary teachers, but it was at Philadelphia [7]. Therefore, this article will describe the result of investigation on the understanding the order of arithmetic operation in Indonesia.

2. Methods

This research used descriptive qualitative approach. Its purpose was describing students' understanding toward the order of arithmetic operation. It involved undergraduate students from mathematics education and elementary school teacher education department. The total number of the participants was 30. The students were selected since they did not learn a specific number sense topic or other subject related to the arithmetic operation at the moment the research was carried out. The students came from three different level year of study.

Data were collected by using a test and questionnaire. The test items were adapted from Billstein [8]. The test consisted of 4 arithmetic questions. The test was used to explore the student's decision in choosing which operation calculated earlier than the others. Each item focused on different order type of operations. The detail type for each item was summarized at Table 1. The test also specifically asked the participants to do one calculation at a time. It was expected to check the order chosen by the students. The test item was displayed as the same order with the order of questions written at first column of Table 1.

Table 1. Types of arithmetic questions

Questions	Type
$3-5\times 6+1$	Multiplication versus Addition and Subtraction
$(3-5)4+3$	Parenthesis
$2-3\times 5+4\times 3-1+3$	Multiplication versus Addition and Subtraction
$7-9\div 3\times 4+5$	Division at the left side of multiplication. Which one comes first?

Furthermore, the questionnaire was used to support the data from the test. It contained 10 questions with true or false model. The questions mostly asked about students' belief and their understanding toward the order of arithmetic operation. These 10 questions summarized at Table 2. Four questions focused on the decision which operation carried out first, while the other six was qualitatively asked students' belief toward the order of typical arithmetic operation.

Table 2. Questionnaire items

Types of question	Questionnaire Item
Choosing the first operation	The first operation for $(3-5)6-1$ is $3-5$
	The first operation for $3-5*6$ is $3-5$
	The first operation for $9 \div 3 \times 4$ is $9 \div 3$
	The first operation for $8-3+7$ is $3+7$
Understanding on the order of operation	Addition ALWAYS be done before subtraction
	Division ALWAYS be done after multiplication
	Operation in the Parenthesis ALWAYS carried out first
	Subtraction is carried out first if it is at the left of addition
	Division is carried out first if it is at the left of multiplication
	Multiplication or division was carried out before addition or subtraction

The data was analyzed by using the qualitative analysis model. The responses collected from the students were displayed on a construct table to investigate the variability occurred on their answers. It used method of describing [9]. The analysis focused on the reason why the students' answer different between each other. It included the decision made by the students that cause the different result of the calculation. This exploration was expected to gather information the typical misconception of the arithmetic operation, in this case was the order, if it existed. This analysis then supported with the responses of the questionnaire.

3. Results and discussion

Data collected from arithmetic test summarized at Table 3.

Table 3. Summary of students' answer

	Test Item			
	1	2	3	4
Correct Answer	25	25	18	23
False Answer	5	5	12	7

It shows the number of students who answer correctly or not for each test items. Around 83% of students can get a correct result of the calculation for test item number 1 and 2. There are 5 students who get a wrong result for those test items. In case of test item 1, the students get incorrect answer not because choosing the subtraction or addition over multiplication. It was only miscalculation. However, students' incorrect response for test item 2 shows different cause. Two students multiply 5 to 4 without realize that 5 belong to the parenthesis. Two other students calculate subtraction at the same time doing addition as shown at Figure 1. It might be a usual error, since the response for questionnaire item 7 got 100% correct. All students agreed that the grouping in the parenthesis was a priority.

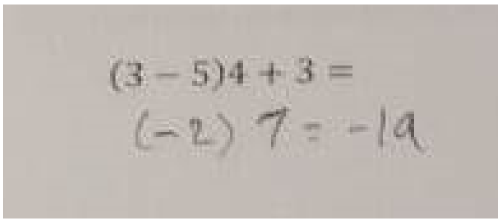

Figure 1. Example of incorrect calculation

In addition, 60% of students answer test item 3 correctly. It was the longest questions in the test. Based on their answer sheets, the calculation was missed because 40% of students carelessly doing the operation. Some of them did not pay attention to the test direction to calculate one operation at a time. For those students who answer correctly, the addition was prioritized than subtraction as seen at Figure 2. This finding was supported by the response for questionnaire item 2 and 10. Both items got 100% positive response. Compare to subtraction or addition, multiplication was carried out first. Choosing addition over subtraction in this case did not imply the calculation result to get wrong.

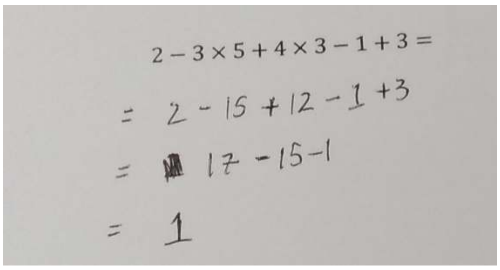

Figure 2. Addition first before subtraction

Interesting finding was found for test item 4. Six out of seven students who got the wrong result calculated multiplication before division. One of these answers was shown at Figure 3. It was rather inconsistent. The response for questionnaire item 3 shows that 95% of students understand that division at operation $9 \div 3 \times 4$ was carried out before the multiplication. This response is consistent with the response for questionnaire item 6 and 9. Since the questionnaire was given after they finished the test. It can be interpreted that the students gain an understanding when they have to give response to the questionnaire. This case was different from case at Figure 2. The inverse of multiplication or division would involve fraction that might lead to the wrong calculation result. However, exploring division as the reciprocal of multiplication may reinforce and help students to gain better understanding on fraction arithmetic [4].

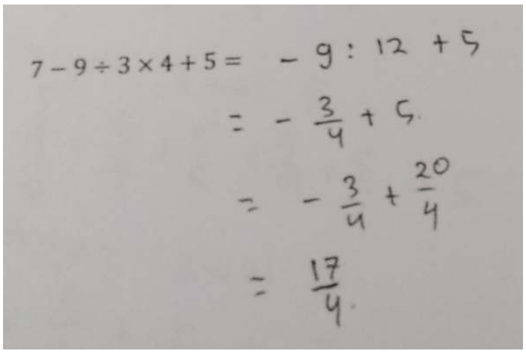

Figure 3. Multiplication before division

Based on all the cases provided from the test and questionnaire, the calculation process was not relying on a strict rule of order. This study did not specifically investigate whether the students know and follow a certain mnemonic. However, students' response of the questionnaire gave raw description on their understanding of the order. Therefore, teaching for understanding the order of arithmetic operation is more important than merely focused on mnemonic such as PEMDAS [6].

4. Conclusion

Although it cannot be generalized, this study contributes to the representation of understanding on arithmetic operation in Indonesia. Prospective teacher for mathematics and elementary school in this study shows their comprehension on the order of the operation. No mnemonic effect that can lead students' understanding to strictly follow the rules of operation order based on the mnemonic. This study highlights the importance of discussing order when learning arithmetic operation. It implies the need to design specific discussion on mathematics classroom about the order of arithmetic operation.

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