Mathematical model of shallot with grandong pest / leafminer

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Abstract. Shallot is the main commodity of Central Sulawesi. But in the cultivation, there are some pests shallot plants. One of the pests is grandong pest/leafminer. In this study, we built a mathematical model of shallot growth with grandong pest/ leafminer. We obtained two critical points, namely A and B. The existence and stability of the two critical points can be guaranteed in this study.

1. Introduction

Central Sulawesi has considerable potential land for vegetable crops, especially shallot. if the production of shallot increases then the income also increases, and vice versa.. Many factors resulted in a decrease in shallot production. One of the factors that affect the decrease of shallot production is pest attack. Grandong (leafminer) is one of the pests that can give a very high loss on the cultivation of shallot.

Many studies has been done about the interaction of shallot and Leafminer. Level of Attack and Type of Leafminer on Three Local Varieties of shallot in Palu Valley of Central Sulawesi [8]. Damage to Chromosomes of shallot (Allium cepa L.) Due to Immersion with Ethidium Bromide [3]. Bioecology and leafmine control Liriomyza Chinensis Kato (Diptera: Agromyzidae) on shallot [7]. Potential for Development and Availability of shallot Leaf Technology in Central Sulawesi [5]. Analysis of Agroecological Zones of Central Sulawesi (Donggala District) [2]. The Effect of Organic and Inorganic Fertilizer Use and Shallot Cultivar Production of Palu [4]. Study of Farmers and Marketing of Patch Shallot [6]. In this study, we will constuct the mathematical model of the interaction of shallot and leafminer population.

The rest of this paper is organized as follows: section 2 is proposed the mathematical model of the interaction of shallot and leafminer population. The stability analysis of critical points are discused in section 3. Section 4 presented the numerical simulation of the mathematical model. Finally, the conclusion is offered in Section 5.

2. Formulation of the Model

Mathematical model of shallot growth with grandong will be constructed in this section. In this study, shallot populations are grouped into 4 compartments (sub-population) based on their growth stages of life, namely the initial growth phase, the Vegetative phase and the generative phase consisting of the tuber formation phase and the maturation phase. Grandong attack shallot almost in all phases of the shallot life. Furthermore, the Grandong populations are grouped according to their life cycle, i.e. eggs, larvae, pupa, and adult. However, in this study, it was assumed that only eggs and larvae of grandong attacked shallot. The compartment diagram illustrating the interaction of shallot and leafminer population can be seen in Figure 1, where the description of the parameter shown in Table 1.

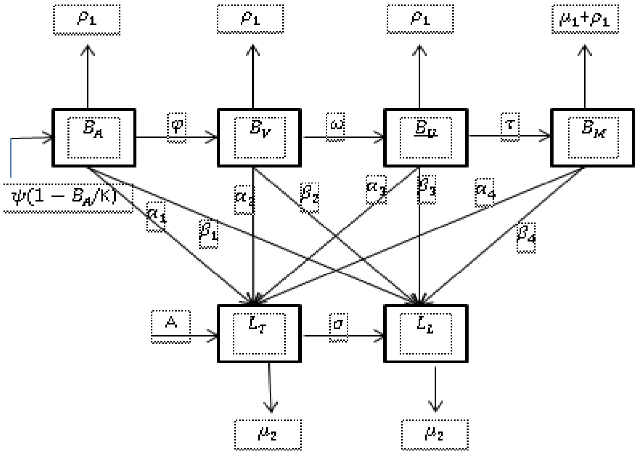

Figure 1. Compartement Diagram

Based on figure 1, the mathematical model of shallot growth with grandong pest/leafminer as follows:

$$\frac{dL_T}{dt} = A - \sigma L_T - \mu_2 L_T \quad (1)$$

$$\frac{dL_L}{dt} = \sigma L_T - \mu_2 L_L \quad (2)$$

$$\frac{dB_A}{dt} = \psi B_A \left(1 - \frac{B_A}{K}\right) - \varphi B_A - \alpha_1 B_A L_T - \beta_1 B_A L_L - \rho_1 B_A \quad (3)$$

$$\frac{dB_V}{dt} = \varphi B_A - \omega B_V - \alpha_2 B_V L_T - \beta_2 B_V L_L - \rho_1 B_V \quad (4)$$

$$\frac{dB_U}{dt} = \omega B_V - \tau B_U - \alpha_3 B_U L_T - \beta_3 B_U L_L - \rho_1 B_U \quad (5)$$

$$\frac{dB_M}{dt} = \tau B_U - \alpha_4 B_M L_D - \beta_4 B_M L_L - (\mu_1 + \rho_1) B_M \quad (6)$$

3. Analysis of the Model

The critical point of the model can be obtained from $\frac{dL_T}{dt} = 0, \frac{dL_L}{dt} = 0, \frac{dB_A}{dt} = 0, \frac{dB_V}{dt} = 0,$ $\frac{dB_U}{dt} = 0, \frac{dB_M}{dt} = 0$. We have two critical points, namely $T_1 = \left(L_T^* = \frac{A}{\sigma + \mu_2}, L_L^* = \frac{A}{(\sigma + \mu_2)(\mu_2)}, B_A^* = 0, B_V^* = 0, B_U^* = 0, B_M^* = 0 \right)$ represented the condition of shallot extinction and $T_2 = \left(L_T^* = \frac{A}{\sigma + \mu_2}, L_L^* = \frac{A}{(\sigma + \mu_2)(\mu_2)}, B_A^* = -\frac{K}{\psi} \left(\varphi + \frac{\alpha_1 A}{\sigma + \mu_2} + \frac{\beta_1 A}{(\sigma + \mu_2)(\mu_2)} + \rho_1 - \psi \right), \right.$

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$$B_V^* = \frac{\varphi\left(-\frac{K}{\psi}\left(\varphi+\frac{\alpha_1 A}{\sigma+\mu_2}+\frac{\beta_1 A}{(\sigma+\mu_2)(\mu_2)}+\rho_1-\psi\right)\right)}{\left(\omega+\alpha_2\left(\frac{A}{\sigma+\mu_2}\right)+\beta_2\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+\rho_1\right)}, B_U^* = \frac{\omega\left(\frac{\varphi\left(-\frac{K}{\psi}\left(\varphi+\frac{\alpha_1 A}{\sigma+\mu_2}+\frac{\beta_1 A}{(\sigma+\mu_2)(\mu_2)}+\rho_1-\psi\right)\right)}{\left(\omega+\alpha_2\left(\frac{A}{\sigma+\mu_2}\right)+\beta_2\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+\rho_1\right)}\right)}{\left(\tau+\alpha_3\left(\frac{A}{\sigma+\mu_2}\right)+\beta_3\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+\rho_1\right)}$$

$$B_M^* = \frac{\dfrac{\tau\omega\left(\frac{\varphi\left(-\frac{K}{\psi}\left(\varphi+\frac{\alpha_1 A}{\sigma+\mu_2}+\frac{\beta_1 A}{(\sigma+\mu_2)(\mu_2)}+\rho_1-\psi\right)\right)}{\left(\omega+\alpha_2\left(\frac{A}{\sigma+\mu_2}\right)+\beta_2\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+\rho_1\right)}\right)}{\left(\tau+\alpha_3\left(\frac{A}{\sigma+\mu_2}\right)+\beta_3\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+\rho_1\right)}}{\left(\alpha_4\left(\frac{A}{\sigma+\mu_2}\right)+\beta_4\left(\frac{A}{(\sigma+\mu_2)(\mu_2)}\right)+(\mu_1+\rho_1)\right)}$$

represented endemic leafminer condition. The existence of the first critical point can be guaranteed since all parameter values are positive. While, T_2 exist if $\psi > \varphi + \frac{\alpha_1 A}{\sigma+\mu_2} + \frac{\beta_1 A}{(\sigma+\mu_2)(\mu_2)} + \rho_1$

Furthermore, the stability of the critical points can be obtained using linearization method. The first critical point is stable if $\psi < \varphi + \mu_1 + \alpha_1 f + \beta_1 g + \rho_1$. While T2 stable if $K < 2h.$, where $h = -\frac{K}{\psi}\left(\varphi + \frac{\alpha_1 A}{\sigma+\mu_2+\rho_2} + \frac{\beta_1 A}{(\sigma+\mu_2+\rho_2)(\mu_2+\rho_2)} + \rho_1 - \psi\right)$

4. Numerical Simulation

The stability of the critical points can be observed through numerical simulation using Matlab software. We give initial values $\left(L_T^{\,0}, L_L^{\,0}, B_A^{\,0}, B_V^{\,0}, B_U^{\,0}, B_M^{\,0}\right) = (300, 150, 5000, 4500, 4320, 4032)$ and parameter values as shown in Table 1. The simulation are shown in figure 2 and 3.

Tabel.1 The parameter values used in the simulation

Parameter	Description	Value	Reference
φ	The transition rate from the B_A compartment to the B_V compartment	0,9	$\frac{Initial\ growth\ population\ of\ shallots}{The\ vegetative\ population\ of\ shallots}$
ω	The transition rate from the B_V compartment to the B_U compartment	0,96	$\frac{The\ vegetative\ population\ of\ shallots}{The\ population\ of\ shallot\ tuber\ formation}$
τ	The transition rate from the B_U compartment to the B_M compartment	0,93	$\frac{The\ population\ of\ shallot\ tuber\ formation}{Shallot\ maturation\ population}$
σ	The transition rate from the L_T compartment to the L_L compartment	0,75	$\frac{Population\ of\ leafminer\ eggs}{Population\ of\ leafminer\ larvae}$

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μ_1	Death shallot death rate naturally	0,067	$\frac{1}{life\ time}$
μ_2	The rate of pest mortality is natural	0,083	$\frac{1}{life\ time}$
ρ_1	The rate of death of shallot caused by pest	0,5	$\frac{1}{life\ time}(leafminer\ population)$
α_1	L_T and B_A interaction rate	0,25	$\frac{Population\ of\ leafminer\ eggs}{Initial\ growth\ population\ of\ shallots}$
α_2	L_T and B_V interaction rate	0,278	$\frac{Population\ of\ leafminer\ eggs}{The\ vegetative\ population\ of\ shallots}$
α_3	L_T and B_U interaction rate	0,289	$\frac{Population\ of\ leafminer\ eggs}{The\ population\ of\ shallot\ tuber\ formation}$
α_4	L_T and B_M interaction rate	0,31	$\frac{Population\ of\ leafminer\ eggs}{Shallot\ maturation\ population}$
β_1	L_L and B_A interaction rate	0,1876	$\frac{Population\ of\ leafminer\ larvae}{Initial\ growth\ population\ of\ shallots}$
β_2	L_L and B_V interaction rate	0,208	$\frac{Population\ of\ leafminer\ larvae}{The\ vegetative\ population\ of\ shallots}$
β_3	L_L and B_U interaction rate	0,217	$\frac{Population\ of\ leafminer\ larvae}{The\ population\ of\ shallot\ tuber\ formation}$
β_4	L_L and B_M interaction rate	0,233	$\frac{Population\ of\ leafminer\ larvae}{Shallot\ maturation\ population}$
A	Constant growth rate L_T	75	Nonci N and Muis A

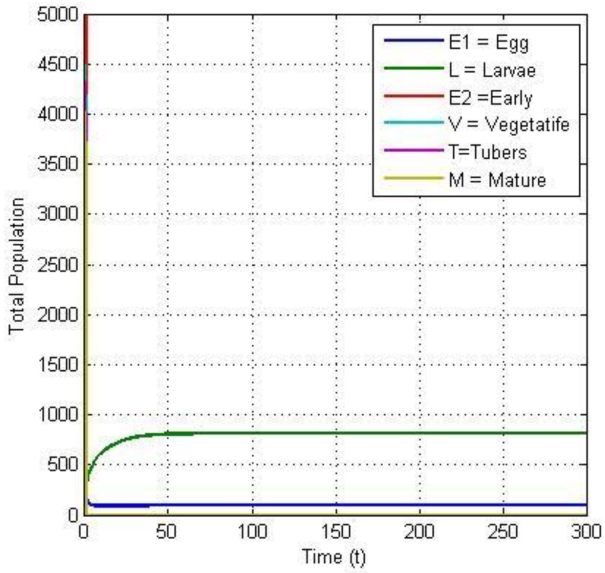

Figure 2. Simulation T_1

Figure 2 shows the shallot population in each phase has decreased and finally tend to zero. This is not an expected condition. The decline is caused by the large level of pest attacks on the shallot. But eggs and larvae of leafminer still exist because of their growth is not dependent on the shallot. The population of leafminer on the egg phase showed a decrease in population. This large decrease is caused by the presence of the transition from egg to larvae and the natural death. While the leafminer population in the larval phase has increased as a result of the transition from egg to larvae.

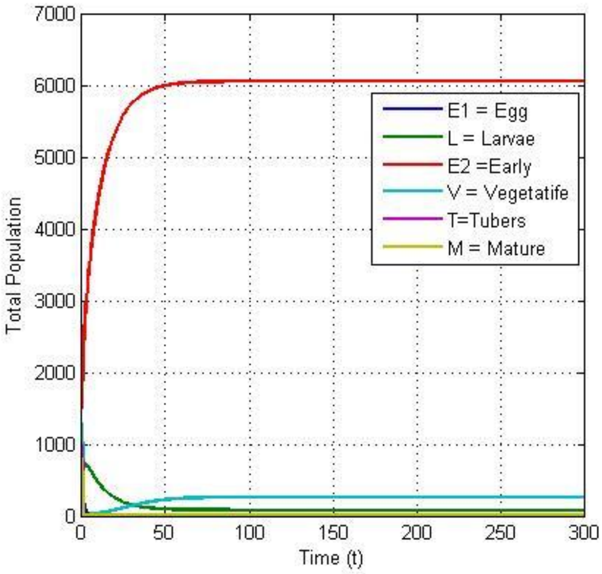

Figure 3. Simulation T_2

Figure 3 shows a decrease of leafminer population in the egg phase due to the transition from egg to larval phase and the presence of natural death. While the leafminer population in the larval phase increased at the beginning of time due to the transition from egg to larval phase. But in the remainder of the time, the population of larvae has decreased due to the phase shift of life into an adult natural death and finally converge to zero. While in the early phase of the shallot increased due to the growth rate of the shallot is bigger than the interaction rate of leafminer and shallot. Furthermore, in the vegetative phase, the formation of bulbs and embankments initially also decreased due to the interaction of leafminer and shallot. Then the vegetative population increased again due to the transition rate from the initial phase to the vegetative phase. While the population of shallots during tuber formation did not increase, although there was a transition from the vegetative phase to the tuber formation phase. The same condition occurs in the maturation phase, where there is no increase in population although there is a transition from the tuber formation phase to the maturation phase. This condition is not ideal because of the leafminer persist and can give an impact on the onion population decline and even run out. Thus pesticides administration to leafminers is needed to control leafminer growth

4. Conclusion

In this study, we have been introduced a mathematical model that describes the interaction of shallot and leafminer population. We have two critical points that represent condition of shallot extinction and endemic leafminer, respectively. The existence and stability. of the two critical points can be guaranteed in this study.

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