Estimation of Value at Risk and Expected Shortfall for Financial Time Series Using a Hybrid GARCH Generalized Pareto Distribution Model

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Abstract. This paper explains a method for estimating Value at Risk (VaR) and Expected Shortfall of heteroscedastic financial return time series. The method used is combination of GARCH models and Extreme Value Theory (EVT). The GARCH models used to estimate volatility and EVT for estimating the tail of distribution. The distribution used in EVT is Generalized Pareto Distribution (GPD). Furthermore, the method used is a method estimation of conditional VaR and conditional Expected Shortfall.

1. Introduction

Risk is something worth to be given an attention. The risk, though very small, is always there so it cannot be eliminated just like that. Therefore, risk measurement needs to be done so that the risk is in a controlled level so as to reduce the occurrence of investment losses [5]. Risk measurement is very important in various fields, especially in the financial field. Both investors and financial institutions are highly vulnerable to risks, especially rare risks, but result in substantial losses. This has led to better risk control measures and methods for measuring these risks [4].

[1] and [2] stated that risk value can be measured in several ways, including Value at Risk (VaR). VaR is the estimated value of the maximum losses that may occur in certain periods with certain confidence levels and in normal market conditions. VaR provides information on the magnitude of the loss, the time period and the level of confidence. However, often the value of the loss exceeds the estimated VaR value. In this case, VaR cannot inform the magnitude of losses in the tail loss, i.e. losses incurred at a level of confidence that exceeds the VaR confidence level. So it was introduced the size of the risk that can explain the value of the loss. The size of the risk in question is Expected Shortfall. Expected Shortfall is often called Expected Tail Loss (ETL). Expected Shortfall is the average of tail loss or loss exceeding VaR at a certain level of confidence.

Financial time series data usually have a tendency to fluctuate rapidly over time so that the variance of errors will always change over time (heteroscedastik). In addition, the financial data usually occur grouping the volatility of gathering a number of errors with a relatively equal magnitude in some time is close (volatility clustering). In calculating VaR and Expected Shortfall values, volatility is important. Because volatility can describe how much deviation that occurs between the expected value and the value of realization or actual value that occurred. One model that focuses on volatility modeling is the Autoregressive Conditional Heteroscedasticity (ARCH) model introduced by Engle (1982). The ARCH model in practice is known to tend to use relatively large parameters (large order) to describe volatility. To avoid high order volatility modeling introduced the Generalized

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Autoregressive Conditionally Heteroscedastic (GARCH) model introduced by Bollerslev (1986) [7] and [8].

Extreme Value Theory (EVT) is a method for determining risk value both with VaR and with Expected Shortfall by trying to determine the distribution of extreme values or events. EVT can also be used to model the extreme events caused by the tail of the distribution of the fat (heavy tail). The methods offered in EVT are Block Maxima (BM) and Peaks Over Threshold (POT), where the resulting distributions are Generalized Extreme Value (GEV) and Generalized Pareto Distribution (GPD) [3].

In this paper research the method used for estimation of VaR and Expected Shortfall is the combination between GARCH and EVT model. EVT method used is POT-GPD.

2. Methods

2.1. Value at Risk

For threshold values u which is great then the distribution function of $F_u(y)$ will approach GPD.

$$G_{\xi,\beta}(y)=\begin{cases}1-(1+\xi y/\beta)^{-1/\xi} & if\ \ \xi\neq 0\\ 1-exp(-y/\beta) & if\ \ \xi=0\end{cases}$$

with $F_u(y)$, that is:

$F_u(y)=P(X-u\leq y|X>u)=\frac{F(y+u)-F(u)}{1-F(u)}$ then the opportunity distribution for $x=y+u$ with the provision of $X>u$, that is:

$$\begin{aligned}F(x)&=\left(1-F(u)\right)F_u(y)+F(u)\\&=\left(1-F(u)\right)G_{\xi,\beta}(y)+F(u).\end{aligned}$$

For value u the big one, $F(u)$ can be calculated with $(n-N_u)/n$ where n is the number of sample observations and N_u is the number of observations greater than u. Therefore, $F(x)$ obtained as follows:

$$\begin{aligned}F(x)&=\left(1-\frac{n-N_u}{n}\right)G_{\xi,\beta}(y)+\frac{n-N_u}{n}\\&=1-\frac{N_u}{n}\left(1+\xi\frac{y}{\beta}\right)^{-\frac{1}{\xi}}.\end{aligned}$$

Next we will look for the quantitative estimation value of the GPD tail distribution or denoted by $\bar{F}(x)=1-F(x)$. For $x=y+u$, then the GPD tail distribution is:

$$\bar{F}(x)=1-F(x)=\frac{N_u}{n}\left(1+\xi\frac{y}{\beta}\right)^{-\frac{1}{\xi}}.$$

Value at Risk (VaR) is the estimated maximum loss of an institution at a certain level of confidence and time range. In the calculation, if known to a return approaching a particular distribution, then VaR is the quantitative of the distribution at a predetermined level of confidence [6]. Mathematically, VaR with a level of confidence α, denoted $q(\alpha)$, expressed as a quartile form to $(1-\alpha)$ of the GPD tail distribution. So the VaR GPD is obtained:

$$q(\alpha)=u+\frac{\beta}{\xi}\left([(n/N_u)(1-\alpha)]^{-\xi}-1\right) \tag{1}$$

In this case, the GPD VaR is an inverse form of the GPD tail CDF. The following model can be obtained for conditional VaR estimation for 1-step:

$$VaR=\mu_{t+1}+\sigma_{t+1}\left(u+\frac{\beta}{\xi}\left([(n/N_u)(1-\alpha)]^{-\xi}-1\right)\right) \tag{2}$$

with μ_{t+1} and σ_{t+1} is a 1-step forecasting result for the mean and volatility functions of heteroskedastic cases.

2.2. Expected Shortfall

Expected Shortfall is a loss that exceeds VaR at a certain level of confidence. Can also be interpreted as the average of tail loss. Expected Shortfall is mathematically written as $E(Loss|\ Loss>VaR)$.

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For a random variable W which distributes GPD with tail parameters $\xi < 1$ and scale parameters β it can be shown that:

$E(W|W > w) = \frac{w\xi+\beta}{1-\xi}$.

where $w\xi + \beta > 0$.

Thus, for the value of Loss exceeding the VaR value, the Expected Shortfall can be obtained:

$$E(Loss|\, Loss > q) = q\left(\frac{1}{1-\xi} + \frac{\beta-\xi u}{(1-\xi)q}\right). \quad (3)$$

So the Expected Shortfall model is conditional:

$$ETL = \mu_{t+1} + \sigma_{t+1}\, q\left(\frac{1}{1-\xi} + \frac{\beta-\xi u}{(1-\xi)\, q}\right). \quad (4)$$

3. Results

3.1. Data

The data used for this case study is AALI daily stock data. Characteristics of data analyzed is data log trading stock returns calculated from the closing price of AALI shares. This stock price data is secondary data obtained from www.finance.yahoo.com.

Below is the time-lapse chart plot and the closing price return of AALI stock trading using EViews software.

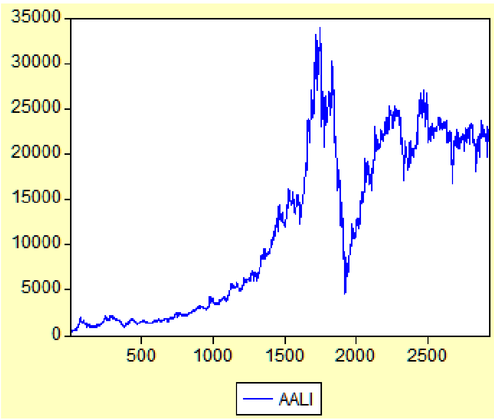

Figure 1. The time series data plot of AALI stock closing price

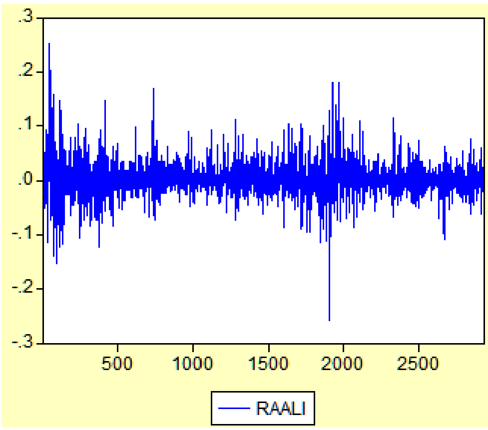

Figure 2. Plot log return data closing price of AALI stock

3.2. GARCH

GARCH is used to model heteroscedastic cases of time serial models. The described time serial model is the variance function of the data. In variance modeling with the GARCH model, modeling of the mean, where the mean modeling and variance modeling is done simultaneously between the two. GARCH modeling analysis is used EViews software help.

After diagnostic checking on some models, it can be concluded that the GARCH (1,1) model is the best model to describe AALI log data return. Below is shown the estimation results for GARCH (1,1) process with constants:

Table 1. Model GARCH (1,1) with constants

Dependent Variable: RAALI
Method: ML - ARCH (Marquardt)
Date: 10/04/12 Time: 19:57
Sample(adjusted): 2 2924
Included observations: 2923 after adjusting endpoints
Convergence achieved after 18 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001489	0.000476	3.128499	0.0018
	Variance Equation			
C	1.17E-05	1.15E-06	10.20035	0.0000
ARCH(1)	0.070459	0.004405	15.99451	0.0000
GARCH(1)	0.919310	0.003836	239.6764	0.0000

R-squared	-0.000060	Mean dependent var	0.001246
Adjusted R-squared	-0.001088	S.D. dependent var	0.031312
S.E. of regression	0.031329	Akaike info criterion	-4.354129
Sum squared resid	2.864968	Schwarz criterion	-4.345945
Log likelihood	6367.559	Durbin-Watson stat	1.792139

Forecasting for the mean function and the volatility function using the best model, the GARCH (1,1) model obtained the mean value 2925 = 0.001489 and the value of the 2925 = 0,033838 volatility.

3.3. Generalized Pareto Distribution

In the calculation of Value at Risk (VaR) and Expected Shortfall with GPD, data return is assumed to have a fat tail (heavy tail). Data with a fat tail distribution generally follow the Pareto distribution or Pareto family. Therefore, it will be tested whether true data return AALI does not follow the normal distribution. Kolmogorov Smirnov test obtained p-value < 0,05, which means H_0 rejected. So it can be concluded that the data return AALI is not normally distributed. Thus, the assumption that the data does not follow the normal distribution is met.

Further testing of GPD effects in the data so that the approach taken in the calculation of VaR and Expected Shortfall really has accommodated the form of empirical data distribution. Because even if data does not follow a normal distribution, it does not mean that data can be inferred to have a heavy tail and GPD distribution. Testing is done by looking at QQ-plot. QQ-plot data where the return data on the extreme number 200 indicates that the data follows the GPD distribution.

The determination of the threshold values of the data in this case study using EasyFit software assistance. In determining the threshold value the extreme amount is limited to 200. The extreme amount is limited to obtain the optimal threshold value. Threshold value obtained, that is $\hat{u} =$ 0,043460.

The GPD parameter estimation model is performed numerically. To simplify the calculation, Matlab program is used. Value obtained $\hat{\xi} = 0{,}138000$ and $\hat{\beta} = 0{,}024070$.

3.4. Value at Risk

From the estimation results obtained GPD parameters, it can then be used in the estimated VaR and Expected Shortfall conditional for AALI shares. Conditional VaR is calculated by the following formula:

$$VaR = \mu_{t+1} + \sigma_{t+1}\left(\hat{u} + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-\alpha)\right)^{-\hat{\xi}} - 1\right)\right)$$

In this case, the conditional VaR estimation uses 95%, 99%, 99.5%, 99.9% and 99.99% confidence levels. The results of the conditional VaR estimation obtained are as follows. Maximum loss for investment of Rp 1 in AALI shares with 95% confidence level is Rp 0,003220 and maximum loss for investment of Rp 1 in AALI shares with 99% confidence level is Rp 0,004753. Furthermore, the maximum loss for investment of Rp 1 in AALI shares with 99.5% confidence level is Rp 0,005526 and maximum loss for investment of Rp 1 in AALI shares with 99.9% confidence level is Rp 0,007632. Then the maximum loss for investment of Rp 1 in AALI shares with 99.99% confidence level is Rp 0.011587.

3.5. Expected Shortfall

Furthermore, the estimated conditional Expected Shortfall estimation is calculated by the following formula:

$$ETL = \mu_{t+1} + \sigma_{t+1} z_q\left(\frac{1}{1-\xi} + \frac{\beta-\xi u}{(1-\xi)z_q}\right)$$

Estimated Expected Shortfall estimates use 95%, 99%, 99.5%, 99.9% and 99.99% confidence levels. The conditional ETL estimation results obtained are as follows. The worst expected loss with a confidence level exceeding 95% for an investment of Rp 1 in AALI shares is Rp 0.004207 and the worst expected loss with a confidence level exceeding 99% for investment of Rp 1 in AALI shares is Rp 0, 005985. Furthermore, the worst expected loss with a confidence level exceeding 99.5% for an investment of Rp 1 in AALI shares is Rp 0.006881 and the worst expected loss with a confidence level exceeding 99.9% for investment of Rp 1 in AALI shares is Rp 0,009324. Then the worst expected loss with a confidence level exceeding 99.99% for investment of Rp 1 in AALI shares is Rp 0,013913.

4. Conclusion

Based on the result of the discussion, this study can be concluded that the estimated value at Risk and Expected Shortfall conditional yield relatively smaller value than the value at Risk and Expected Shortfall unconditional. Value at Risk and Expected Shortfall is conditional enough to be used because it accommodates the form of data return distribution that has a heavy tail and time varying variance.

5. References

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