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# Improvement of grey model for predicting crack growth

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**Abstract**. Predicting crack growth of any structure is a prerequisite for reliable and effective structural health prognostics. This paper presents a model to predict the future state of crack growth based on grey model and one-step-ahead forecasting technique. Grey model is a time series forecasting model that uses operations of accumulated generation to construct differential equations. Specifically, the feasibility of grey model as a predictor for crack growth prognostics system has been investigated. Basic Grey model has employed to forecast the future state of crack growth, but the result was unsatisfactory. In order to improve the accuracy of prediction, a modification of Grey model has been developed. Finally, real trending data of crack testing was employed for evaluating the modified model. However the model was built by using a few input data, it is able to track closely the change of crack growth path.

## 1. Introduction

Cracks may occur in the structures caused by some common factors such as temperature, stresses, corrosion, and manufacturing faults. Nevertheless, the growth of crack induces to inevitable fractures which may cause failure. Nowadays, various techniques have been developed and used to predict crack growth such as grey model based technique.

Grey model has been applied successfully in various area. Hsu and Chen [1] applied grey model for predicting power demand. The use of grey system theory in predicting the road traffic accident was introduced by Mohammadi et al [2]. Subsequently, Gu [3] employed grey prediction model for predicting failure of electronics. On the other hand, Tangkuman and Yang [4] introduced application of grey model for machine degradation prediction.

This paper presents grey model application for predicting crack growth. Grey model theory, which was originally proposed by Deng [5], is able to effectively deal with incomplete data for system analysis, modeling, prediction, decision making, and controlling. In a grey model, its information is neither totally clear as in a white system nor totally unknown as in a black system. Grey model sets each stochastic variable as a grey quantity that change within a given range. They deal directly with the original data and search the intrinsic regularity of data [6].

In this work, the prediction algorithm has been developed is grey model coupled with one-step-ahead technique. Number of cycle and crack length data is acquired to validate the prediction model. This proposed method is addressed to know the crack length in the future. Observing the result of crack length prediction will know whether the component needs to be replaced or not.

## 2. Methodology

### *2.1. Basic grey model GM (1,1)*

The grey forecasting model uses the operations of accumulated generation to construct differential equations. Intrinsically speaking, it has the characteristics of requiring less data. The grey model GM(1,1), i.e., a single variable first-order grey model, is summarized as follows [5,7]:

Step 1: For an initial time sequence,

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(i), \dots , x^{(0)}(n)\} \tag{2}$$

where $x^{(0)}(i)$ denotes the time series data at time ith.

Step 2: On the basis of the initial sequence $X^{(0)}$, a new sequence $X^{(1)}$ is set up through the accumulated generating operation in order to provide the middle message of building a model and to weaken the variation tendency, i.e.

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)(}i), \dots , x^{(1)}(n)\} \tag{3}$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \qquad k = 1, 2, \dots, n \tag{4}$$

Step 3: The first-order differential equation of grey model GM(1,1) is then the following

$$\frac{X^{(1)}}{} \qquad {}^{(1)} \tag{5}$$

and its difference equation is

$$x^{(0)}(k) + aZ^{(1)}(k) = b \qquad k = 2, 3, \dots, n \tag{6}$$

and from Eq. (5), it is easy to get

$$\begin{matrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \\ \lfloor x^{(0)}(n) \rfloor \end{matrix} \quad \begin{matrix} Z^{(1)}(2) & \\ Z^{(1)}(3) & \\ & \\ \lfloor -Z^{(1)}(n) & 1 \rfloor \end{matrix} \quad [\ ] \tag{7}$$

where a and b are the coefficients to be identified. Let

$$Y_n = [x^{(0)}(2), x^{(0)}(3), \dots , x(^{0)}(n)]^T \tag{8}$$

$$\begin{matrix} Z^{(1)}(2) & \\ Z^{(1)}(3) & \\ & \\ \lfloor -Z^{(1)}(n) & 1 \rfloor \end{matrix} \tag{9}$$

where $Y_n$ and $B$ are the constant vector and the accumulated matrix respectively. Also take

$$Z^{(1)}(k+1) = ½( x^{(1)}(k) + x^{(1)}(k+1)) \qquad k = 1, 2, \dots, (n-1) \tag{10}$$

where $Z^{(1)}(k+1)$ is the $(k+1)$th background value. And

$$A = [a, b]^T \tag{11}$$

Applying ordinary least-square method to Eq. (7) on the basis of Eqs. (8) – (11), coefficient $A$ becomes

$$A = (B^T B)^{-1} B^T Y_n \tag{12}$$

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*Step 4:* Substituting $A$ in Eq. (6) with Eq. (12), the approximate equation becomes the following

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - b/a)\, e^{-ak} + b/a \tag{13}$$

where $\hat{x}^{(1)}(k+1)$ is the predicted value of $x^{(1)}(k+1)$ at time $(k+1)$. After the completion of an inverse accumulated generating operation on Eq. (13), $\hat{x}^{(0)}(k+1)$, the predicted value of $x^{(0)}(k+1)$ at time $(k+1)$ becomes available and therefore,

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{14}$$

*2.2. Modification of grey model*

Basically, grey model GM(1,1) is able to fit well equidistance and slow growth time sequences [8]. Moreover, Mao and Tan [8,9] suggested an improved method based on the background value of grey model GM(1,1) in Eqs. (9-10). Revising these equations have been proved to be a successful way of widening the adaptability of grey model GM(1,1) to various kinds of time sequences.

In this work, the improve equation is then given as

$$Z^{(1)}(k \quad 1) = (\frac{1}{\quad}) \times ((1 \qquad (k)) \qquad (k \quad 1)) \tag{15}$$

## 3. Application and result

In order to validate the prediction model, experiment data from a website was employed in this work [10]. This data contains number of cycles and crack length information based on a crack testing. The prediction model is addressed to know the crack growth from a crack testing.

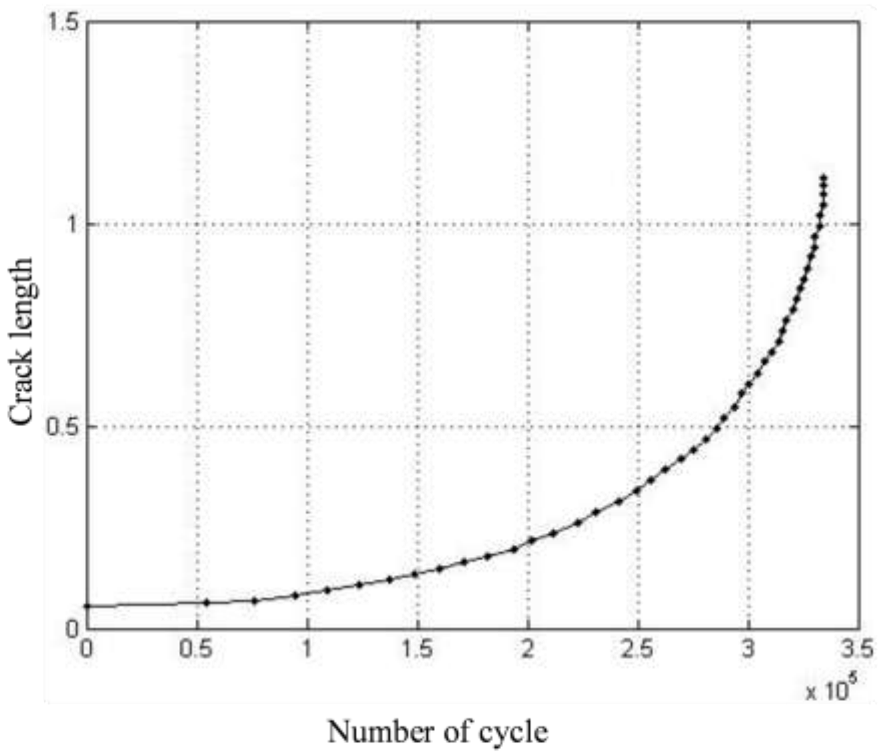


**Figure 1.** Crack testing data

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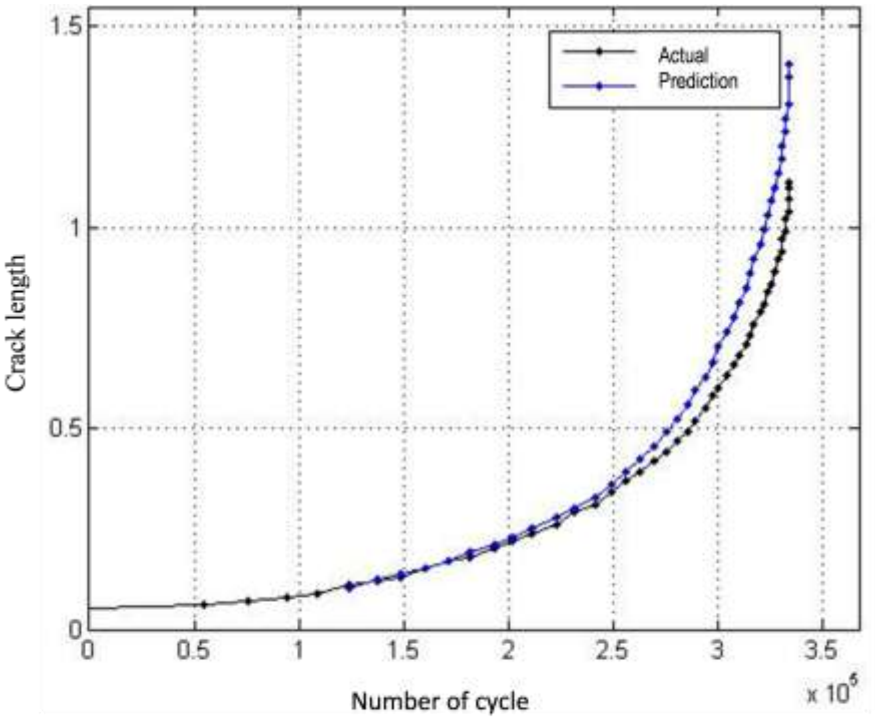


**Figure 2.** Actual and prediction of crack growth – before modification

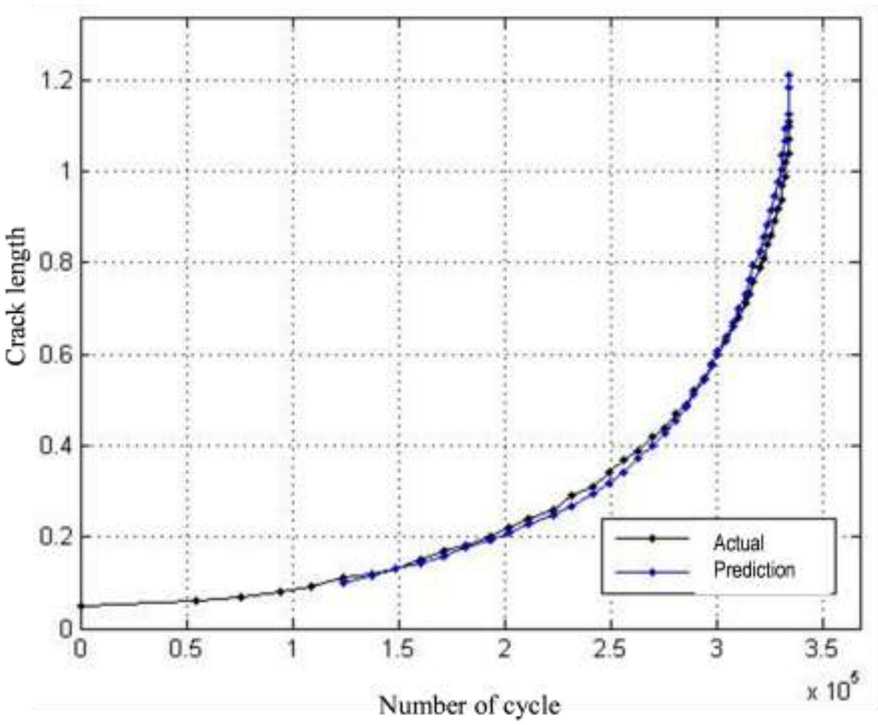


**Figure 3.** Actual and prediction of crack growth – after modification

Using grey model and one-step-ahead technique the future state of crack growth could be predicted as shown in Fig. 2. The prediction result indicates that the model which cannot reach well the actual values. Although the model is able to predict the trend of crack growth, the model cannot detect well actual values at several points. Moreover, the prediction could not reach closely the actual final length of the crack. Therefore, modifying the grey model is needed to increase the accuracy of prediction.

Eventually, Fig. 3 presents the prediction of crack growth after modification. The accuracy of the modified grey model is very satisfying; the modified model can anticipate accurately crack length in the future. Before modification, the prediction model has different percentage of 14 %. On the other hand after modification, the prediction model has different percentage of 4,89 %. Therefore, there is an improvement different percentage of 9,11 %. However the model was built by using a few input data, it is able to track closely the change of crack growth path.

## 4. Conclusion

The prediction model was validated by predicting crack length in the future from a crack testing. The model was able to track closely the changes of crack growth that has different percentage of 4,89 %. The feasibility of grey model as a predictor for crack growth prognostics system has been investigated. Specifically, the result indicates that the proposed model may be a potential predictor for crack growth prognostics.

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