Thales Theorem in Indonesian Lower Secondary Textbooks

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Abstract. Textbook is an important part in teaching and learning process. Furthermore, it is geting more attention recently. However, how mathematics textbook provide more opportunity in learning a specific mathematics content, e.g Thales Theorem has not been extensively studied. We consider the Anthropology theory of the didactic (ATD) as a theoritical aproach, particularly the notion of didactic transpotition between scholarly knowledge and knowledge to be taught. The purpose of this study is to analyse how Thales Theorem is discussed in university textbook and school textbook. We studied four university textbooks and seven online Indonesian lower secondary textbooks that are authorised by the ministry of education. The result shows how the explanations of Thales theorem in the university textbooks have more variation than the explanation in school textbook. Contrarily, school textbook uses a monoton way in discussing Thales theorem.

1. Introduction

The glory of Greek geometry has left traces which still remain on the school mathematics. For instance, In Indonesian lower secondary school, students still encounter an application of the idea of Thales theorem. Thales theorem is a generalization of the basic principle of similarity that use for measuring the height of a pyramid. Nevertheless, that principle rarely used with explicit reference to Thales theorem. In this study, we consider Euclid's element as relevant theorem. In the book VI (2), it said that If a straight line be drawn parallel to one of the sides of the triangle, it will cut the sides of triangle proportionality which means the ratio of dividing part of two sides are the same.

Normally, Thales theorem is located in similarity theme in lower secondary school textbooks in Indonesia. From this theorem, students are given a propotionality formula '$\frac{a}{b} = \frac{c}{d}$' from two similar triangle in 'Thales triangle' as can be seen in figure 5. Then students are asked to find a missing number of correspondence sides of similar triangle. There are textbooks that provide where the proportionality formula come from. However, we also found textbooks that do not provide the explanation of Thales theorem. At the same time.

Based on aforementioned above we can conclude that Thales theorem is an important theme but it is not well discussed in textbooks. Thus, a textbooks analysis is needed.Textbook analysis research has been growing significantly. However, an analysis textbook research using ATD is still rare. García (2005) offered more complete analysis approach for proportionality task. In his thesis, he conducted a research about bridging proportion in arithmetics domain and algebra domain in lower secondary textbooks using ATD. He found that there is a poor connection between two domains. This research also brings a new horizon in textbook analysis whereas ATD can be used to analyse a mathematical context across domains. ATD can also be used to analyse a historical study, Hersant (2005) conducted a textbook analysis and focused on missing value task in ratio and proportion in arithmetic domain. In the missing value task, she found six different techniques that can be used to investigate lower secondary textbooks in France in five periods of time from 1887-2004. These six different techniques are reduction unity, multiplication by ratio, proportion, cross product, multiplication by ratio, and graphic solution. Her research shows that every era uses different solution due to educational need. This research presents how ATD can be used for textbook comparative analysis between current textbooks and historical textbooks. Even though, it is only used for one particular type of task (missing value problem).

The aim of this study is to analyse the description of the idea of Thales' theorem in the scholarly knowledge (university textbooks) and knowledge to be taught (school textbooks). Additionally, we try to describe the relation between them. To analyse the textbooks, we used Anthropology Theory of Didactic. We mainly focused on the technology of the explanation of the idea of Thales' theorem. It is not our aim to discuss the issue of how Thales theorem should be explained in the current Indonesia lower secondary mathematics textbooks. However, we hope that this study will contribute to the consideration of choosing a textbook for teacher or people in education.

2. Theoritical Framework

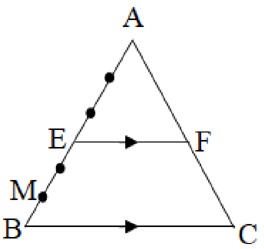
According to the Anthropology Theory of the didactic, knowledge is developed in institutions (Chevallard 1985; Chevallard & Bosch 2014). There are three main steps of institution: scholarly mathematics, knowledge to be taught, and knowledge actually to be taught (Barbe, 2005). The minimum unit of analysis using ATD is describing the process of didactic transposition (Bosch & Gascon, 2006). This study explains the transposition of Thales' theorem from scholarly knowledge, such as university textbooks to the knowledge to be taught, for instance official school textbooks.

The basic element of anthropology model of mathematical activity called praxeological organisation. The word praxeology stands for praxis and logos. The discourse about praxis relates to the word practice which is built in two levels: type of tasks and technique to solve them. Additionally, the discourse about logos pertains to the word knowledge which is used to interpret and justify the praxis blocks. This discourse is constructed by technology and theory. The praxis blocks corresponding to the type of tasks are quite similar in the textbooks so we focus on the element of theoretical explanations that are or would be given in the textbooks. This motivates our discussion in the next section.

3. The Scholarly Knowledge

we propose four university textbooks from different year as a resource for scholarly knowledge (Wenthworth (1899), Betz & Webb (1912), (Ford & Ammerman, 1920), Ringenberg & Presser (1971)). we will describe the variation of the proofs of Thales' theorem based on line segment, the area of triangles, and vectors. We will consider proof for the slight reformulation that in the situation if figure 1, one has $EB : AE = FC : AF$.

3.1. Line segment proof



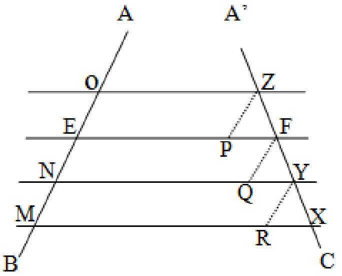


Fig. 1. Proving using commensurable side. Fig. 2. Parallel lines intercepts equal parts

Following the proof by Wenthworth (1899, p 177), we will consider a triangle which has commensurable sides (see figure 1). MB is an example of common measure AE and EB. Let AB be contained m times in EB and n times in AE. Then, $EB : AE = m : n$. From each common measurement point of AB, draw lines parallel to BC intersect AC. If parallel lines intercepts equal parts on one transversal, they will intercepts equal parts on every transversal for instance ZF, FY, YX on the transversal $A'C$ (see figure 2). Following Wenthworth (1899, p 59), we can draw $ZP, FQ, YR \parallel$ to AB

Then $\angle OEF, \angle ENY, \angle NMX$ are equal (*interior angles*)

$\therefore \angle ZPF, \angle FQY, \angle YRX$ are equal (*interior angles*) …(1)

And $\angle PZF, \angle QFY, \angle RYZ$ are equal (*interior angles*) …(2)

Also $ZP = OE, FQ = EN, YR = NM$ (*parallels comprehended parallels are equal*)

$\therefore ZP = FQ = CQ$ (*parallels comprehended parallels are equal*) …(3)

Based on (1), (2), and (3), having two angles and one side of each respectively equal, we can get $\Delta ZFP = \Delta FYQ = \Delta YXR$, then $ZF = FY = YX$. The proof of parallel lines regarding figure 2 reveals that parallel lines from each common measurement in figure 2 will divide FC into m and AF into n.

Then , $EB : AE = m : n$ and $FC : AF = m : n$

$\therefore EB : AE = FC : AF$.

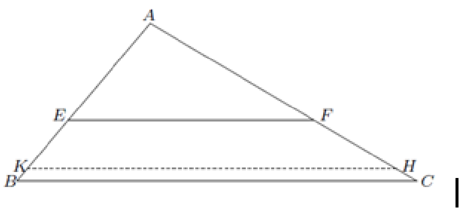


Figure 3. Proving using Incommensurable sides

When AE and EB cannot be divided in to equal part, if we take unit length m times in AE and apply it to EB, there will remain length KB which will be less than m. Whatever the choice of m, we have by case $EB : AE = FC : AF$. As m is taken smaller and smaller. Then, EK comes closer to KB, while FH comes closer to HC. By taking sufficiently small, we can thus bring AE / EK and AF / FH to the respective values AE / EB and AF / FC. Consequently, these last ratios differ from the preceding equal ratios. This means that they are actually equal. Thus we must have, as was to be proved $AE / EB = AF / FC$ (Ford & Ammerman, 1920).

3.2. Area of triangle proof

Following discussion is proving the theorem using area of triangle based on Betz & Webb (1912, p 240), Ringenberg & Presser (1971, p. 409) (see figure 3).

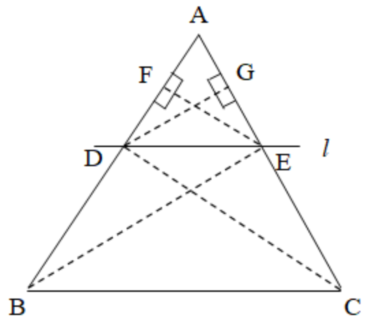


Figure 4. Proving theorem using area of a triangle

In ΔABC, a straight line $l \parallel BC$, intersect AB at D and AC at E, then $\frac{AD}{DB} = \frac{AE}{EC}$

Proof: link BE and CD. Draw $EF \perp AB$ and $DG \perp CA$.

EF is the height of ΔADE and ΔDBE

$$\frac{Area\Delta ADE}{Area\Delta DBE} = \frac{\text{½ AD.EF}}{\text{½ DB. EF}} = \frac{AD}{DB} \quad \ldots (1)$$

$$\frac{Area\Delta ADE}{Area\Delta ECD} = \frac{\text{½ AE.DG}}{\text{½ EC. DG}} = \frac{AE}{EC} \quad \ldots (2)$$

ΔDBE and ΔDCE are on the same base DE and between the same parallel straight line BC and DE. So that, area ΔDBE =area $\Delta ECD \ldots$(3).

From (1), (2) and (3) we can conclude $\frac{AD}{DB} = \frac{AE}{EC}$.

3.3. Vectors proof

We will present how we can combine vectors to prove Thales' theorem (see figure 4).

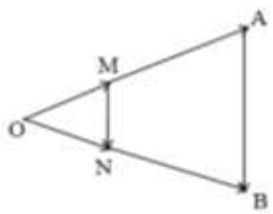


Figure 4. Proving theorem using vectors

It is formulated as in the theorem of figure 4, one has

$$\overrightarrow{MN} \parallel \overrightarrow{AB} \Leftrightarrow \frac{\|\overrightarrow{OB}\|}{\|\overrightarrow{OM}\|} = \frac{\|\overrightarrow{OA}\|}{\|\overrightarrow{ON}\|}$$

Proof:

$\Rightarrow$ $\overrightarrow{MN} \parallel \overrightarrow{AB}$ means that there is $k \in \Re$ such that $\overrightarrow{AB} = k\overrightarrow{MN}$

from this We have l, m so that

$$\left.\begin{aligned} \overrightarrow{OB} &= l\overrightarrow{ON} \\ \overrightarrow{OA} &= m\overrightarrow{OM} \end{aligned}\right\} \ldots (*)$$

$$\overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN}$$

$$\overrightarrow{OM} = \overrightarrow{ON} - \overrightarrow{MN}$$

$$\begin{aligned} \overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= m\overrightarrow{OM} + k\overrightarrow{MN} \\ &= m(\overrightarrow{ON} - \overrightarrow{MN}) + k\overrightarrow{MN} \\ &= m\overrightarrow{ON} + (k-m)\overrightarrow{MN} \end{aligned}$$

Know that $\overrightarrow{OB} = l\overrightarrow{ON}$

$$l\overrightarrow{ON} = m\overrightarrow{ON} + (k-m)\overrightarrow{MN}$$

$$(l-m)\overrightarrow{ON} = (k-m)\overrightarrow{MN} \ldots (**)$$

But $\overrightarrow{ON}$ and $\overrightarrow{MN}$ are not parallel so (**) implies that $l = m$ and $k = m$

Therefore (*)

$$\frac{\|\overrightarrow{OB}\|}{\|\overrightarrow{ON}\|} = \frac{\|k\overrightarrow{ON}\|}{\|\overrightarrow{ON}\|} = k, \quad \frac{\|\overrightarrow{OA}\|}{\|\overrightarrow{OM}\|} = \frac{\|k\overrightarrow{OM}\|}{\|\overrightarrow{OM}\|} = k$$

$\overrightarrow{OB} \parallel \overrightarrow{ON}, \overrightarrow{OA} \parallel \overrightarrow{OM}$ there is l, m so that $\overrightarrow{OB} = l\overrightarrow{ON}$ and $\overrightarrow{OA} = m\overrightarrow{OM}$ then

$$l = \frac{\|\overrightarrow{OB}\|}{\|\overrightarrow{ON}\|} = \frac{\|\overrightarrow{OA}\|}{\|\overrightarrow{OM}\|} = m$$

Therefore $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= l\overrightarrow{ON} - l\overrightarrow{OM} \\ &= l(\overrightarrow{ON} - \overrightarrow{OM}) \\ &= l\overrightarrow{MN} \end{aligned}$$

So $\overrightarrow{AB} \parallel \overrightarrow{MN}$

4. Context and Knowledge to be Taught

The first paragraph after a heading is not indented (Bodytext style). The idea of Thales' theorem appears in the third grade of Indonesian lower secondary school. We used five textbooks which are available online in the http://bse.kemdikbud.go.id/. Additionally, these books are approved by the ministry of education (see table 1). The letter G in the table 1 symbolizes that the idea of Thales' theorem is located in the geometry topic called similarity. Uniquely, it reveals that the three proofs in the scholarly knowledge do not appear in those five textbooks. Granted that these proofs are too complicated for students becomes a reason for authors to avoid these proofs. Also, these proofs are far from similarity topic in which the idea of Thales' theorem is located. In the next section, we will discuss the technology of this theorem in the five textbooks.

Table 1. List of online textbooks

	Book title	Authors, year of publication
G_1	Book for studying mathematics 3	A. Wagiyo, Sri Mulyono, Susanto (2008)
G_2	Contextual Teaching and Learning Mathematics	R Sulaiman, Tatag Yuli Eko S, Toto Nusantara, Kusrini, Ismail, Atik wintarti (2008)
G_3	Easy way to learn Math 3	Nunik Avianti Agus (2008)
G_4	Active and enjoyable learning in math 3	Wahyudin Djumanta and Dwi Susanti (2008)
G_5	Mathematics 3	Masduki and Icwhan Budi Utomo (2007)

4.1. Technology

Among five textbooks, only one textbook (G3) which does not discuss this theorem. Generally, Thales' theorem is not stated clearly, but it is only used to solve certain tasks. For example, students are asked to prove that (see figure 5).

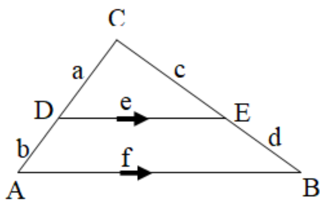


Figure 5. Proving theorem using similarity

These four books use the same technology to prove $\frac{a}{b} = \frac{c}{d}$. Firstly, the authors discusses about connecting AAA with similarity. By having the same measurement of corresponding angles, students can determine the similarity of triangles (definition 1). Using another triangle similarity definition, students can address the proportionality measurement of corresponding sides (definition 1). Consequently, similarity has two definitions in this task.

$$\left.\begin{array}{l} \angle CDE = \angle CAB\,(corresponding\ angle)\ \ldots\ (1) \\ \angle CED = \angle CBA\,(corresponding\ angle)\ \ldots(2) \\ \angle DCE = \angle ACB\,(joint\ angle)\ \ldots(3) \end{array}\right\} def.1$$

Based (1), (2), and (3), $\Delta DCE \sim \Delta ACB$

$\Delta DCE \sim \Delta ACB, then\ \frac{CD}{CA} = \frac{CE}{CB} = \frac{DE}{AB}$, def 2

(Wagiyo, Mulyono & Susanto, 2008).

To learn similarity, it is started by measuring the corresponding angles and corresponding sides of polygon. The purpose of this activity is to reveals that two similar polygons have a proportional measure of corresponding sides and same measure of corresponding angle. Triangle is a special case of polygon. To prove triangle, students are also asked to do some measurement activity. For instance:
Given two triangles (see figure 5), Djumanta & Susanti (2008) discuss he activity to proof similarity. By assuming that $e // f$, measure the side of CD, DA, CE, EB, DE, AB and measure the angle of DCE, ACB, CDE, CAB, CED, CBA. Based on the measurement result, We will find:

$$\frac{CD}{DA} = \frac{CE}{EB} = \frac{DE}{AB}; \ldots(1)$$

$$DCE = ACB, CDE = CAB, CED = CBA \ldots \text{ (2)}$$

According to the (1) and (2), given two triangles, if the corresponding angles are the same measure, then the corresponding sides are proportional. This means that given two triangles, if the corresponding side are proportional, then the two triangles are similar. Reciprocally, if the corresponding sides are proportional, then the corresponding angles are the same measure. In another word, given two triangles, if the corresponding angles are the same measure, then the two triangles are similar. From the explanation above, we can conclude that, that two triangles are similar, if the corresponding side are proportional or corresponding angles has the same measure

Another way to prove the similarity is using a scale factor. In Masduki and Utomo (2007), the students are asked to draw their first triangle with their own measurement. Then, by using scale factor, students decide their own second triangle. Using the same instruction, the students are asked to measure the corresponding angle and corresponding sides. Again, students are facing the same condition as activity before that the corresponding angles are the same measure, then the corresponding sides are proportional and the vice versa. Then authors finish the activity by writing the definition of similarity.

As can be seen that textbooks provide a measurement activity for students to prove similarity. Using numbers to prove is very compromising. However, students will miss mathematical reasoning part. Students prove the similarity by encounter special case and get the general fact. However, to think mathematically, students are encouraged to experience from general fact. For two similar polygons, students are having one definition: *two similar polygons have a proportional measure of corresponding sides and the same measure of corresponding angle*. Therefore, students face two definitions of two triangle similarity:

(1) Given two triangles, if the corresponding angles are the same measure, then the two triangles are similar

(2) Given two triangles, if the corresponding sides are proportional, then the two triangles are similar

It is something unusual that one thing has two definitions. One of them must be stated as the effect or corollary. However, to prove similarity, especially triangle similarity using deductive reasoning is not an easy thing for students. Moreover, the authors want to emphasize student’s technique skill rather than reasoning skill.
Secondly, to prove, the authors are using the algebraic definition:

$$\Delta DCE \sim \Delta ACB \text{ then } \frac{CD}{CA} = \frac{CE}{CB} = \frac{DE}{AB} \text{ or } \frac{a}{a+b} = \frac{c}{c+d} = \frac{e}{f}$$

By choosing $\frac{a}{a+b}=\frac{c}{c+d}$

$\Leftrightarrow a(c+d)=c(a+b)$

$\Leftrightarrow ac+ad=ac+bc$

$\Leftrightarrow ad=bc$

$\Leftrightarrow \frac{a}{b}=\frac{c}{d}$

Based on the explanation above, even though this proof appears in the geometric domain called similarity, we can see that how big algebraic domain influenced dominantly and explained perfectly in these four books. The algebraic domain is well defined and student will easily understand.

Relating to the transposition, we can see that knowledge actually to be taught disconnect to the scholarly knowledge. It is because the big influence of curriculum. Also, we can see that the textbooks avoid using Thales's theorem, but this textbook use the idea of Thales's theorem.

5. Conclusion

We have pointed out a result based on the discussion above. The explanation of Thales' theorem in the university textbooks use line segment, area of triangle and vectors, but school textbooks use similarity topic. Moreover, school textbooks proof has a lot influenced from algebraic domain in which this theorem appears in the geometry domain. To discuss similarity, students are asked to do some measurement activity. Then, they get the definition of similarity. Student concludes the definition from empirical case to the general case. However, the idea of mathematical reasoning is from a general case of the specific case. Furthermore, students also face two kinds of triangle similarity definition which very unusual in mathematics world.

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